Liquidity, trading size, and the coexistence of dealership and auction markets *

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Abstract

This paper analyses the coexistence of two markets for the same shares, a quote-driven market and an order-driven market, as observed for example for the trading of continental shares on the London SEAQ International. The focus is on the trade-off between the uncertain execution price faced by investors on an auction market and the implicit transaction cost represented by the spread in a dealer market. We obtain that those investors who desire to make large trades will prefer to trade with the dealer, while trades of smaller size will be carried out on the auction market. Moreover, we explicitly investigate the interrelations between the two markets showing that the pricing policy followed by a dealer depends on the conditions prevailing on the auction market.

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1. Introduction

This paper provides a theoretical analysis of the parallel working of two markets for the same asset (equity), differing in their trading structure. On the one side we consider an order-driven market organised as an auction. A distinguishing feature of this system is execution-price uncertainty: when submitting their orders, investors do not know the price that will prevail in the auction. On the other side we consider a quote-driven system, operated by dealers who post firm prices that are publicly known before orders are submitted. The dealer stands ready to buy or sell the commodity at the quoted prices, earning a return generated by the spread between the selling (or ask) price and the buying (or bid) price. The two markets radically differ in the degree of "liquidity" offered, where liquidity is meant as (i) the ability to sell or buy equities immediately and without significant price variations and, (ii) the ability to carry over trades at low transaction costs. By focusing on the choice of risk-averse investors between these two alternative trading mechanisms, we show that the relative advantages and disadvantages of each system can be seen as a trade-off between price uncertainty on the auction market and the transaction cost implicit in the bid-ask spread set by market makers operating the dealership market.

The analysis of the interaction between auction and dealership markets may help to interpret some of the most recent developments in European stock markets. In 1986, after the launch of the SEAQ International quote-driven trading system, London seemed likely to become the chief marketplace for blue-chip Continental stocks. This trend was reversed during the 1990s, following the radical modifications which affected the main Continental exchanges. Markets such as the Paris Bourse, the Madrid Bolsa, the Borsa Valori in Milan replaced their old-fashioned batch auctions with continuous, electronic order-driven systems. These changes have sharply reduced -but not eliminated- the trading volumes of Continental stocks handled by SEAQ dealers. According to Steil (1996) and Pagano (1997), European exchanges have been converging to a common dualistic structure, where automated auction systems coexist together with a dealership segment. Pagano (1997) also singles out two main stylised facts about the parallel working of such different trading structures. First, auction systems have specialised in small and medium-sized trades, while the SEAQ dealership market has been devoting almost exclusively to large trades. Second, there seems to be both interdependence and competition between the SEAQ International and the Continental bourses. With regard to markets' interdependence, London dealers exploit European exchanges to rebalance their positions in foreign shares. At the same time, the evidence suggests that Continental markets exert competitive discipline on London dealers who quote prices on the same stocks.

In the present model, we explicitly consider the opportunity for risk-averse traders to operate in either of two stock markets characterised, in a very stilized way, as an auction market where agents have to bear the risk of an uncertain price, and a dealer market where the advantage of firm prices is obtained at the cost of the positive spread which is monopolistically set by the dealer. We model price uncertainty by assuming that the auction market works as a batch auction. Although Continental auction markets have recently moved to countinuous systems based on limit-orders, the batch auction hypothesis does not constitute a major limitation to our setup. Indeed, also limit-orders remain associated with a large price-uncertainty on order execution.[1]

Our framework builds on the model proposed by Pagano (1989), where investors subject to endowment shocks choose between two alternative auction markets possessing the same trading technology. Each investor is non-atomistic, in the sense that her trade decisions have some influence on the auction price, and chooses the optimal stock of the asset on the basis of her conjecture about the demand of the other agents participating in the auction, as in Kyle (1985). Here, differently from Pagano (1989), investors can choose between two different trading technologies for the same stocks: an auction market and a dealership market, where the market-maker quotes firm ask and bid prices.

We represent the interaction between dealer and investors as a three-stage dynamic game. In the first stage, a (monopolistic) dealer sets the prices at which he is ready to buy and sell stocks. In the second stage, investors decide whether to operate in the dealer market at the quoted prices or in the auction market, according to their individual endowment shock, and trading with the dealer occurs. In the third stage, finally, orders are executed in the auction market. This sequential structure is motivated by the fact that, when investors decide whether to place their orders on the auction market, they can generally observe the spreads currently quoted by dealers. Further, the timing we assume allows the dealer to participate in the auction market to rebalance her position in the stock.

Our framework does not require frictions such as asymmetric information to generate its main results. Although adverse selection problems are central in most of the literature on trading structures (see, among many others, Madhavan (1992) and Pagano and Roell (1996)), our model builds exclusively on "execution risk", which is, the risk of adverse price changes when the order is fulfilled. As emphasised in some recent theoretical and empirical literature, transaction costs on auction markets seem to be consistently lower than the costs imposed by dealers'

[1] As argued by Penati (1993), limit-orders "do not necessarily guarantee the full execution of an order and, even less ... the price at which the order will be executed". This is why the existence od dealers can provide "certainty about the order execution conditions ... to institutional investors involved in block-trading".

markets (see De Jong, Nijman and Roell (1995)). In the model of Pagano and Roell (1996), dealership markets are characterised by a low degree of transparency on executed trades. As a consequence, dealers are forced to quote high spreads to protect themselves from informed traders. As Pagano and Roell (1996) note, however, their model "...fails to explain why some traders still prefer trading in dealer markets. For example, despite the increasing popularity of the competing auction markets, London's dealers still retain a clientele of wholesale international equity dealing". Their suggestion, which we try to capture in the present paper, is that "agents who are averse to execution risk might prefer the implicit insurance offered by the dealer market" (p.598).

The market microstructure literature has analyzed the parallel working of markets for the same asset. In most cases, however, the trading structures are assumed to be the same. The model in Pagano (1989) builds on the investors' choice between two auction markets possessing the same trading technology, showing that the two markets cannot coexist unless there are (exogenously given) differences in transaction costs. Chowdhry and Nanda (1991) analyze a model with informed agents where a security is traded on multiple locations simultaneously. The trading technology is the same in all the alternative markets. Again, the authors find that trade tends to concentrate on a single market. In contrast with these papers, we show that different trading structures, such as the auction and the dealer market, can coexist. Further, we obtain that large traders will prefer the dealer's market, consistently with the facts observed by Pagano (1997). Several papers have compared the performance of dealership and auction markets. Pithyachariyakul (1986), Madhavan (1992) and the fore mentioned Pagano and Roell (1996) contrast auction and dealership markets with regard to their welfare properties, price efficiency, volatility, transparency and liquidity, etc. These contributions, however, do not analyze the conditions under which these market structures can coexist. Interestingly, Seppi (1997) models a hybrid market system, like the NYSE, where a monopolistic market-maker (the "specialist") executes incoming trades in the face of competition from limit orders made by liquidity suppliers. Seppi shows that (i) limit orders severely constrain the exercise of market power by the specialist and, (ii) large traders will prefer a hybrid system to a pure limit order market, since the presence of a specialist guarantees unlimited liquidity for large buys. Seppi's results have a flavour similar to ours. In our model, the equilibrium spread set by the dealer crucially depends on the conditions (participation, average trade size, etc.) of the auction market. This is consistent with the stylised fact according to which Continental exchanges seem to discipline London dealers' behaviour.

Our model is presented in the following section and solved in section 3, where we derive the prices set by the dealer, the conditions under which an investor will choose to trade on either market and the conditions for the coexistence of the two trading mechanism, and discuss the properties of the coexistence equilibrium. The final section summarises our main conclusions.

2. Description of the model

The economy is composed of $N+1$ agents, one risk-neutral dealer and N risk-averse investors,[2] allocating their portfolios between a safe asset and risky equities. The timing of events is as follows (see Figure 1). At time 0, the dealer operates as a monopolist,[3] setting the ask and bid prices (p_A and p_B respectively, with $p_A \geq p_B$) at which he is prepared to sell and buy any amount of shares the investors are willing to trade. At time 1, investors irreversibly choose whether to trade their stocks in the dealer market at the quoted prices, or in the auction market. Whereas in the former prices are firm and known, the execution price in the auction market is uncertain. At time 2, orders are executed on the auction market and stocks are exchanged at the market clearing price p_M . Afterwards, dividends on the stock and interest on the safe asset are paid. Each investor i ($i = 1, ..., N$) maximizes a standard mean-variance objective function in terminal wealth, w_i, with respect to her demand for the stock K_i:

$$E(U_i) = E(w_i) - \frac{b}{2} var(w_i) \tag{2.1}$$

where

$$w_i = dK_i + R\left[w_{0i} + p_J(e_i - K_i)\right] \tag{2.2}$$

and w_{0i} and e_i are the initial endowments of the safe asset and of the stock, respectively, d is the uncertain dividend per share, R is the return to the safe asset, and p_J ($J = M, A, B$) is the price at which the investor exchanges. There are two stochastic elements in the model: the dividend d, distributed with mean μ and variance σ^2, and the endowment disturbances e_i's. We assume that the endowment shocks e_i's are distributed in such a way that for N_α agents e_i may take only the two values $\pm\alpha$ with equal probabilities, while for the remaining N_β agents it may take only the equally likely values $\pm\beta$, with $\alpha > \beta$.[4] These

[2] The assumption that the market maker is risk-neutral while traders are risk-averse is common in the literature: see, for example, Madhavan (1992).

[3] Similarly to Pithyachariyakul (1986) and Seppi (1997) we model the dealer as a monopolistic price-setter. Moreover, Dutta and Madhavan (1997) analyze a dynamic game of inter-dealer competition and show that market-makers are likely to collude implicitly on monopoly prices, especially when the trading-frequency is high. On the role of market power in market making, see also Leach and Madhavan (1993).

[4] A similar assumption is made by Easley and O'Hara (1987), where investors place either large or small orders.

assumptions on the distribution of the endowment shock e allow us to distinguish between two different types of agents: those of "type-α", with relatively large (positive or negative) endowment shocks, and those of "type-β", with relatively small (positive or negative) endowment disturbances. The ratio $\frac{\alpha}{\beta}$ then provides a measure of the degree of heterogeneity among investors that is useful in the discussion of the model's equilibrium properties.

As in Madhavan and Smidt (1993), the dealer operates both as a market maker who provides liquidity on demand by quoting firm prices, and as an investor trading on his own account. At time 1 he sells Q_A shares at the price p_A and buys Q_B shares at p_B. When the auction market opens (time 2), his endowment is equal to the net position in shares resulting from his trading activity at time 1: $(Q_B - Q_A)$. This feature of our setup is consistent with the observation that London dealers rebalance their positions in foreign stocks on Continental markets (see Pagano (1997)). Exchange in the auction market then determines his terminal position in the stock, K_d. The objective function of the (risk-neutral) dealer is:

$$E(U_d) = E(w_d) \tag{2.3}$$

where

$$w_d = dK_d + R\{w_{0d} + p_M[(Q_B - Q_A) - K_D] + p_A Q_A - p_B Q_B\} \tag{2.4}$$

As shown in equation (2.4), the trading at time 1 affects dealer's wealth in two ways: it modifies his wealth allocation between the safe asset and the stock, and it adds profits from the dealership activity to total wealth.

The solution of the model is obtained by working backwards, starting from time 2. At time 2, when orders on the auction market are executed , the investors who chose (at time 1) to operate there and the dealer submit their buying and selling offers, which determine the market-clearing price. The number of agents participating in the auction market is commonly conjectured to be N_M. As already noticed, while investors are hit by stochastic disturbances e_i, the dealer derives his position in the stock from hos market-making activity at time 1. At time 1, investors decide whether to trade immediately with the dealer, or in the auction market (at time 2) on the basis of: i) the observed realization of their own endowment shock e_i, ii) the known prices that the dealer is quoting, and iii) the expected market-clearing price p_M. When an investor evaluates the opportunity to trade in the auction market, she forecasts the price p_M by conjecturing the demand functions of both the dealer and the other $N_M - 1$ traders who are expected to make the same choice. At time 0, the dealer sets p_A and p_B to maximize his indirect utility function, subject to the expected demand functions Q_A and Q_B. To form expectations on the quantity of shares demanded, the dealer needs to have some priors on the number of investors who will choose to buy from and sell to him and on their average trades. In the next sections we discuss these three steps in greater detail.

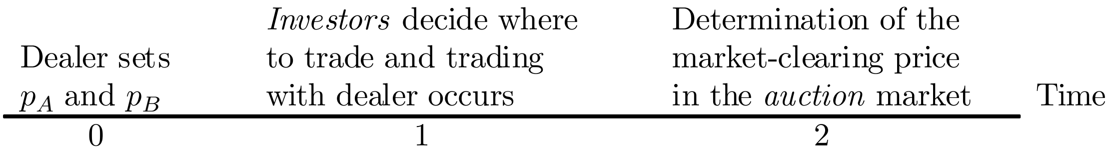

Figure 1: Timing of events

3. The solution of the model and the equilibrium.

3.1. The determination of the market-clearing price.

The procedure used to determine the equilibrium of the auction market closely follows Pagano (1989). Agents calculate the equilibrium in the auction market conjecturing that the residual market demand function is linear (the details of the derivation are in the Appendix). In equilibrium such conjecture is fulfilled.

The actual demand function of the N_M investors who chose to operate in the auction market (whose set is denoted by Φ_M), and of the dealer are, respectively:

$$K_i = \frac{2(N_M-1)}{2N_M-1}\left(\frac{\mu - Rp_M}{b\sigma^2}\right) + \frac{1}{2N_M-1}e_i \qquad i \in \Phi_M \tag{3.1}$$

$$K_d = \frac{2(N_M-1)N_M}{2N_M-1}\left(\frac{\mu - Rp_M}{b\sigma^2}\right) + (Q_B - Q_A) \tag{3.2}$$

By equating total demand, i.e. the N_M actual demands K_i's plus the dealer's actual demand K_d, to total supply, we can compute the equilibrium value of the market-clearing price:

$$p_M = \frac{\mu}{R} - \frac{b\sigma^2}{2R}e_M \tag{3.3}$$

where $e_M = \frac{1}{N_M}\sum_{i\in\Phi_M} e_i$. Were the agents risk-neutral ($b = 0$), the equilibrium price p_M would be equal to the "fundamental" value $\frac{\mu}{R}$. As a consequence of risk-aversion, the higher the average endowment shock e_M, the lower the equity price: when agents receive (on average) large endowments of risky stocks, they tend to reduce the quantity of equity in their portfolios, and therefore depress, in the aggregate, the price p_M.

3.2. Investors' choice between the auction and the dealer market.

At time 1, investors choose whether to trade in the auction or in the dealer market by comparing the expected utility they can attain in the two cases. Considering the case of trading in the *auction market* first, we note that when the outcome of this market is realized, at time 2, the indirect utility function of the investors who chose to operate there is:

$$\begin{aligned} u_i^M &= \mu\, e_i + R\, w_{0i} - \frac{b\sigma^2(3N_M^2 - 3N_M + 1)}{2(2N_M - 1)^2}\, e_M^2 \\ &\quad - \frac{b\sigma^2}{2(2N_M-1)^2}(e_i - e_M)^2 - \frac{b\sigma^2(2N_M^2 - 2N_M + 1)}{(2N_M-1)^2}\, e_M\,(e_i - e_M) \end{aligned} \tag{3.4}$$

When deciding at time 1 where to trade, each investor i is supposed to know only her own disturbance e_i and the first and second moments of the distribution function of the shocks hitting investors operating in the auction market, but not their actual realizations. Given her conjecture about N_M, investor i predicts that:

$$\begin{aligned} E\left(e_M \mid e_i\right) &= \frac{1}{N_M}\, e_i\ ,\ E\left(e_M^2 \mid e_i\right) = \frac{1}{N_M^2}\, e_i^2 + \frac{N_M - 1}{N_M^2}\, \sigma_M^2 \\ E\left(e_M\, e_i \mid e_i\right) &= \frac{1}{N_M}\, e_i^2 \end{aligned} \tag{3.5}$$

where σ_M^2 is the variance of the shocks of investors trading in the auction market. By using (3.5), the expected indirect utility function of investor i subject to trading in the auction market is equal to:

$$E\left(u_i^M \mid e_i\right) = R\, w_{0i} + \frac{b\sigma^2}{2}\left[\frac{2\mu}{b\sigma^2}\, e_i - \frac{1}{N_M}\, e_i^2 + \left(\frac{N_M - 1}{2N_M - 1}\right)^2 \frac{\sigma_M^2}{N_M}\right] \tag{3.6}$$

The three terms in the square bracket in (3.6) have the following interpretation. The first one, involving e_i, is independent of N_M and captures the increase in expected utility due to a larger realization of the endowment shock. The other two terms have the same interpretation given in Pagano (1989). The term in e_i^2 measures the *liquidity value* of the auction market. With non-atomistic agents, the need for large portfolio reallocations - following large endowment disturbances - entails a reduction of the expected utility that is inversely related to the number of participants in the market: for small values of N_M, large trades tend to affect adversely the transaction price p_M.[5] The last term, containing σ_M^2, captures the

[5] A similar effect arises in Seppi (1997) when large trades are considered. Adverse price effects are also present in models with risk-neutral agents and adverse selection: see, e.g., Chowdry and Nanda (1991).

speculative value of the agent's participation in the auction market. When the variance of the shocks hitting the investors trading in the auction market is large, also price volatility is large. As a consequence, each participant has better chances of buying low and selling high, with a positive effect on her expected utility. On the other hand, a large number of market participants tends to reduce this effect by decreasing the average endowment volatility and hence p_M's volatility: as (3.3) shows, the variance of p_M depends exclusively on the variance of e_M, i.e. $\frac{\sigma_M^2}{N_M}$.

In order to determine investors' utility from trading in the *dealership market* we need to derive their "demand" functions. When trading with the dealer, investor i faces a price p_A when buying, and p_B when selling. It follows that her objective function (2.1) is now simply:

$$E(U_i) = \mu K_i + R\left[w_{0i} + p_J(e_i - K_i)\right] - \frac{b\sigma^2}{2}K_i^2 \tag{3.7}$$

with

$$p_J = \begin{cases} p_A & \text{if } e_i < K_i \\ p_B & \text{if } e_i > K_i \end{cases}$$

The optimal demand K_i obtained by maximizing (3.7) can take three values, depending on the realization of the disturbance e_i:

$$K_i = \begin{cases} \frac{\mu - Rp_A}{b\sigma^2} & \text{if} \quad e_i < \frac{\mu - Rp_A}{b\sigma^2} \\ e_i & \text{if} \quad \frac{\mu - Rp_A}{b\sigma^2} \leq e_i \leq \frac{\mu - Rp_B}{b\sigma^2} \\ \frac{\mu - Rp_B}{b\sigma^2} & \text{if} \quad e_i > \frac{\mu - Rp_B}{b\sigma^2} \end{cases} \tag{3.8}$$

Substituting each of the values of (3.8) into (3.7) we obtain the indirect utility function of investor i subject to trading in the dealership market:

$$u_i^D = \begin{cases} \frac{(\mu - Rp_A)^2}{2b\sigma^2} + Rw_{0i} + Rp_A e_i & \text{if} \quad e_i < \frac{\mu - Rp_A}{b\sigma^2} \\ \mu e_i - \frac{b\sigma^2}{2}e_i^2 + Rw_{0i} & \text{if} \quad \frac{\mu - Rp_A}{b\sigma^2} \leq e_i \leq \frac{\mu - Rp_B}{b\sigma^2} \\ \frac{(\mu - Rp_B)^2}{2b\sigma^2} + Rw_{0i} + Rp_B e_i & \text{if} \quad e_i > \frac{\mu - Rp_B}{b\sigma^2} \end{cases} \tag{3.9}$$

As can be seen from the second line of (3.9), when the endowment shock is relatively small, the investor will prefer *not* to trade with the dealer. Indeed, the advantage of shifting away part of the risk involved in her endowment is overweighed by the trading cost implicit in the spread set by the market maker.

On the basis of (3.9) we are now ready to describe investors' choice among the different markets.

The Investor's Rule (*I-Rule*)
For the price vector (p_A, p_B) set by the dealer, and for a given conjecture on N_M, investor i's decision rule in terms of the realized disturbance e_i is:

i) for $N_M \le 1$, no trade occurs in the auction market and the investor trades in the dealership market according to (3.8);

ii) for $N_M > 1$, investor i chooses to:

- buy from the dealer at price p_A if $e_i \le f(p_A,\, N_M,\, \sigma_M^2)$;
- trade in the auction market at price p_M if $f(p_A,\, N_M,\, \sigma_M^2) < e_i < g(p_B,\, N_M,\, \sigma_M^2)$;
- sell to the dealer at price p_B if $e_i \geq g(p_B,\, N_M,\, \sigma_M^2)$,

where:

$$
\begin{aligned}
f(\underset{-}{p_A}, \underset{-}{N_M}, \underset{-}{\sigma_M^2}) &= \left(\frac{\mu - R\,p_A}{b\sigma^2}\right) N_M - \sqrt{\left(\tfrac{N_M-1}{2N_M-1}\right)^2 \sigma_M^2 + N_M(N_M-1)\left(\tfrac{\mu - Rp_A}{b\sigma^2}\right)^2} \\
g(\underset{-}{p_B}, \underset{+}{N_M}, \underset{+}{\sigma_M^2}) &= \left(\frac{\mu - R\,p_B}{b\sigma^2}\right) N_M + \sqrt{\left(\tfrac{N_M-1}{2N_M-1}\right)^2 \sigma_M^2 + N_M(N_M-1)\left(\tfrac{\mu - Rp_A}{b\sigma^2}\right)^2}
\end{aligned}
$$

The rule follows directly by comparing the expected utility in the auction market with the analogous expressions for either buying or selling in the dealership market. Note that also in the case $N_M = 1$, no trade occurs in the auction market, because from (3.1) the excess demands of the dealer and of the only investor in this market are null.

The effect of dealer's prices p_A and p_B on investors' decision is straightforward: with N_M and σ_M^2 constant, a higher p_B and a lower p_A (resulting in a lower spread) tend to narrow the range of the endowment shocks inducing agents to trade in the auction market. A rise in N_M increases the *liquidity* in the auction market and, by reducing the adverse effect of large individual transactions on p_M, tends to attract investors who need large portfolio reallocations. This effect is only partially offset by the fact that a larger N_M reduces the *speculative* value and then the attractiveness of the auction market (with σ_M^2 constant). Overall, an increase in N_M allows traders operating in the auction market to benefit from greater liquidity while avoiding the transaction cost arising from the spread: as a result, a larger value of N_M widens the range of endowment shocks that induce agents to participate in the auction market. Finally, with N_M constant, a higher value of σ_M^2 increases the *speculative* value of the auction market, widening the range of disturbances e_i's for which investors prefer to trade in that market.

3.3. The dealer's price setting problem.

The last step is to characterize the dealer's price setting decision at time 0 and the corresponding equilibria. Given a conjecture on the number of investors willing to trade on each side of the market at the quoted prices, the total quantities of shares sold and bought by the dealer (Q_A and Q_B respectively) are calculated by aggregating the individual excess demands in (3.8):

$$\begin{aligned} Q_A &= \sum_{i\in\Phi_A}(K_i - e_i) = \sum_{i\in\Phi_A}\left(\frac{\mu - Rp_A}{b\sigma^2} - e_i\right) \\ &= \left(\frac{\mu - Rp_A}{b\sigma^2} - e_A\right)N_A \qquad (3.10) \\ Q_B &= \sum_{i\in\Phi_B}(e_i - K_i) = \sum_{i\in\Phi_B}\left(e_i - \frac{\mu - Rp_B}{b\sigma^2}\right) \\ &= -\left(\frac{\mu - Rp_B}{b\sigma^2} - e_B\right)N_B \qquad (3.11) \end{aligned}$$

where $e_A = \frac{1}{N_A}\sum_{i\in\Phi_A} e_i$ and $e_B = \frac{1}{N_B}\sum_{i\in\Phi_B} e_i$; Φ_A and Φ_B denote the sets of the agents buying from and selling to the dealer, respectively, and N_A and N_B the number of these agents.

The dealer's indirect utility function, conditional on the outcome of the auction market, is:

$$u_d = \mu(Q_B - Q_A) + Rw_{0d} + (p_A Q_A - p_B Q_B)R + \frac{b\sigma^2(N_M - 1)N_M}{2(2N_M - 1)}\, e_M^2 \qquad (3.12)$$

At time 0 the dealer sets the ask and bid prices to maximize his expected indirect utility with respect to p_A and p_B, subject to the expected demand functions (3.10) and (3.11). Since the outcome of the auction is unknown at this stage, the unconditional expectations of e_M and e_M^2 are equal to 0 (since all investors have a zero-mean endowment shock) and $\frac{\sigma_M^2}{N_M}$, respectively (where σ_M^2 has been already defined as the variance of the shocks hitting agents trading in the auction market). Hence, the expectation of u_d taken at time 0 is:

$$E(u_d) = (\mu - Rp_B)EQ_B - (\mu - Rp_A)Q_A + Rw_{0d}\frac{b\sigma^2(N_M - 1)}{2(2N_M - 1)}\,\sigma_M^2 \qquad (3.13)$$

Given the investor's rule (*I-Rule*), the quantities traded with the dealer, Q_A and Q_B, depend on their own price only; thereby the ask-side problem separates from the bid-side one. In principle, to determine the utility-maximizing price vector $(p_A,\ p_B)$, the dealer should take three effects into account. The first is the standard demand relationship: the higher p_A (the lower p_B), the lower is the quantity of shares sold (bought) by the dealer to those trading with him. The other two effects stem from the possibility that the investors move to the auction market: on the one side, higher p_A's and lower p_B's may cause Q_A and Q_B to fall further by discouraging investors from choosing the dealership market; on the other side, the migration of marginal investors to the auction market may raise N_M and σ_M^2 (because of their higher endowment shocks relative to those of the investors who had already chosen that market), and thereby the utility of the dealer.

Maximization of (3.13) is carried out by the dealer under the commonly-shared conjecture on N_M (the number of agents trading in the auction market) and N_D (the number of investors choosing the dealership market), which acts as a constraint on his pricing decision: the price vector $(p_A,\ p_B)$ must be such that the N_D investors are satisfied to choose the dealership market on the basis of the investor's rule.

Depending on the (common) conjectures about the number of agents trading in either market, two kinds of equilibria may arise: a *coexistence* equilibrium and a *non-coexistence* equilibria. In what follows, we mainly concentrate on the coexistence equilibrium, where the dealer and the auction market operate together. This equilibrium is of particular economic interest, according to the evidence reported in Pagano (1997).

3.4. The coexistence equilibrium.

The following proposition provides a characterization of the coexistence equilibrium:

Proposition 1. There exists a *coexistence equilibrium* where type-α investors choose the dealership market ($N_D = N_\alpha$), type-β investors choose the auction market ($N_M = N_\beta$) and the dealer maximizes his profits, if the following condition holds

$$\frac{\alpha}{\beta} > 1 + h(N_\beta) \tag{3.14}$$

where:[6]

$$h(N_\beta) \cong \frac{N_\beta - 2}{2N_\beta - 1} + \frac{1}{4N_\beta^2 - 1}\sqrt{4N_\beta^4 + 9N_\beta^2 + 5N_\beta + 1}$$

[6] The expression for $h(.)$ reported here (an approximation of the true -and more cumbersome- function) is *more* restrictive than the condition necessary for the coexistence equilibrium to exist. The function $h(.)$ rapidly decreases towards zero for $N_\beta \geq 2$, so that the condition in (3.14) is easily met.

The equilibrium prices in the dealership market are:

$$p_A^* = \frac{\mu}{R} + \frac{b\sigma^2}{R}\left[\alpha - \sqrt{\frac{N_\beta}{N_\beta+1}\alpha^2 + \frac{N_\beta^2}{(N_\beta+1)(2N_\beta+1)^2}\beta^2}\right] \tag{3.15}$$

$$p_B^* = \frac{\mu}{R} - \frac{b\sigma^2}{R}\left[\alpha - \sqrt{\frac{N_\beta}{N_\beta+1}\alpha^2 + \frac{N_\beta^2}{(N_\beta+1)(2N_\beta+1)^2}\beta^2}\right] \tag{3.16}$$

Proof. In order to have type-α investors trading with him, the dealer must set prices such that for each of them the expected utility is not lower than the expected utility she could get from moving to the auction market. As in the latter case the number of investors in the auction market would be $N_\beta+1$, i.e. all type-β agents plus the investor i moving there, the prices (p_A^*, p_B^*) set by the dealer must satisfy the two inequalities:

$$-\alpha \;\square\; f\left(p_A^*,\, N_\beta+1,\, \beta^2\right) \qquad \text{and} \qquad \alpha \geq g\left(p_B^*,\, N_\beta+1,\, \beta^2\right) \tag{3.17}$$

where $f(.)$ and $g(.)$ are defined above. On the other hand, type-β agents will not move away from the auction market if:

$$-\beta > f\left(p_A^*,\, N_\beta,\, \beta^2\right) \qquad \text{and} \qquad \beta < g\left(p_B^*,\, N_\beta,\, \beta^2\right) \tag{3.18}$$

The solution given by unconstrained maximization of (3.13) is never consistent with the given conjecture: the prices $p_A = \frac{\mu}{R} + \frac{b\sigma^2}{2R}\alpha$ and $p_B = \frac{\mu}{R} - \frac{b\sigma^2}{2R}\alpha$ obtained from $\frac{\partial E(u_d)}{\partial p_A} = 0$ and $\frac{\partial E(u_d)}{\partial p_B} = 0$ violate the constraints (3.17). Hence, the prices chosen by the dealer are found by imposing the equality sign in (3.17), yielding (3.15) and (3.16). Note that, in this case: $p_A^* < \frac{\mu}{R} + \frac{b\sigma^2}{2R}\alpha$ and $p_B^* > \frac{\mu}{R} - \frac{b\sigma^2}{2R}\alpha$. Finally, to ensure that type-β investors actually choose the auction market, the prices (p_A^*, p_B^*) must satisfy also the inequalitities (3.18). By plugging (3.15) and (3.16) into (3.18), we find condition (3.14). ■

In the coexistence equilibrium "large" traders accept to pay the spread to the dealer in return for the full liquidity available at the quoted prices, whereas "small" investors prefer to operate in the auction market at a price that is scarcely sensitive to their trades.[7] As implied by Proposition 1, the presence of the auction market constrains the dealer to choose a corner solution, setting ask and bid prices that make each large trader indifferent between trading with the market-maker,

[7] The opposite situation, where "small" investors trade with the dealer while "large" ones operate in the auction market (i.e. $N_D = N_\beta$ and $N_M = N_\alpha$), can never be an equilibrium since the analogues of (3.17) and (3.18) cannot be simultaneously satisfied.

or moving to the auction market. If auction market competition were absent, the dealer would be free to set prices which solve his problem as an internal solution.

From the equilibrium prices in (3.15) and (3.16) we easily derive the spread S^* charged by the dealer:

$$S^* \equiv p_A^* - p_B^* = \alpha \frac{2b\sigma^2}{R} \left[1 - \sqrt{\frac{N_\beta}{N_\beta + 1} + \frac{N_\beta^2}{(N_\beta + 1)(2N_\beta + 1)^2} \left(\frac{\beta}{\alpha} \right)^2} \right] \tag{3.19}$$

Although only type-α investors, needing relatively large portfolio reallocations, trade on the dealership market, the size of the (utility-maximizing) spread set by the dealer depends on parameters that characterise trading on the auction market, where only type-β investors operate. For instance, the number of type-β investors is a crucial determinant of the overall "value" of the auction market, and thereby affects the alternative utility level that type-α agents can attain by leaving the dealer market. Thus, in order to attract large traders, the dealer sets a spread which accounts for the conditions on the alternative market. If the number of traders is large, the liquidity value of the auction market increases, since the adverse effect of any individual transaction on the market price is reduced. Although the speculative value of the market is diminished, since a large number of participants reduces price variability as shown by the second term under the square root in (3.19), in our set-up the liquidity effect dominates, determining an inverse relationship between N_β and the size of the dealer's spread S^*. Moreover, an increase in the size β of the endowment shock of the agents operating in the auction market increases the speculative value of that market also for type-α investors, with a consequent reduction in the spread necessary to induce trading. Finally, when the endowment disturbance α increases, widening the degree of heterogeneity among agents, the auction market becomes a less attractive alternative for sizeable trades. The dealer can thus exploit the greater need for liquidity of "large" investors by setting a higher spread. The finding that the auction market exerts "competitive discipline" on the dealer's behaviour is consistent with the evidence. As reported in Pagano (1997), the London SEAQ's market "touch" on cross-listed stocks is consistently wider when Continental exchanges are closed.[8]

The model also allows for *non-coexistence* equilibria where only the auction market, or the dealership market, survives.

Proposition 2. There are rational conjectures that can support *non-coexistence equilibria.*

[8] As noted by Pagano (1997), the presence of informed traders provides another possible reason, besides the "competitive discipline" argument, as why the market "touch" is wider when Continental markets are closed. Indeed, SEAQ dealers may rely on the guidance of the prices realised on Continental exchanges to reduce adverse selection problems.

We break the discussion of Proposition 2 into two cases.

(i) Only the dealership market operates. If agents commonly conjecture that auction market participation is sufficiently scarce (i.e. $N_M \ll 1$), a *dealership equilibrium* will arise (see Appendix 2 for further details). In this case, the spread may be so high to exclude type-β agents from trading in equilibrium. Note also that if the prices quoted by the dealer are such that conditions (3.18), and hence (3.14) do not hold, the only rational conjecture is that no type-β agent will participate in the auction market, that is $N_\beta = 0$. Thus, violation of (3.14) leads to an equilibrium where only the dealership market operates.

(ii) Only the auction market operates. When agents conjecture that everybody will trade in the auction market (i.e. $N_D = 0$), the dealer will expect to trade a quantity equal to zero on both the bid and ask side ($EQ_A = EQ_B = 0$), and thus he will abstain from quoting a spread. In this case, only the auction market will operate.

4. Conclusions

When the participation in an auction market is limited, large trades can generate relevant price variations. As a consequence, a risk-averse agent who would like to carry on a large transaction has to take into account the adverse price effect she induces on a thin market. On the contrary, in a dealer market, market makers quote firm prices at which they buy and sell shares, eliminating the adverse price effect at a cost given by the spread. This is the basic concept underlying the model presented. The natural implication we obtain is that investors who are willing to carry on large transactions are mainly concerned with the adverse price effect they induce on the auction market: as a consequence, they will prefer to trade with the dealer, bearing the cost represented by the spread wedge. On the contrary, trading on the auction market will be preferred by investors needing small portfolio reallocations for two reasons. First, the expected adverse price effect induced by a small transaction does not justify the payment of the cost implicit in the spread. Second, a trader who does not need relevant portfolio readjustments has the opportunity of buying at relatively low prices and selling at relatively high prices in the auction market, by exploiting its trade-driven price volatility.

Our model, thus, not only allows for the coexistence of the two trading structures in equilibrium, but it also generates predictions on the sorting of trade size in each market that are consistent with observation. Indeed, the SEAQ International dealer market has become increasingly specialized in large trades, while the Continental auction markets seem to have a comparative advantage in the retail segment (see Pagano (1997), Pagano and Roell (1996)). Further, the interrelation between the working of the two markets constitutes a salient feature of the present paper. By rebalancing his position in foreign stocks, the dealer affects the auction market price. Moreover, we have shown that the conditions of the auction market (in terms of participation and average trade size) impose "competitive discipline" on the pricing policy of the market-maker. For this reason, our model provides some testable empirical implications about the size of the spread set by the dealer when a quote-driven and an order-driven market operate in parallel. In particular, the spread in the dealer market should be inversely related to the alternative "value" offered by an auction market, as measured, for example, by participation.

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Appendix 1.

As in Pagano (1989), the N_M investors who chose to operate in the auction market and the dealer conjecture that the residual market demand function have the following linear form:

$$\sum_{h\in\Phi_M-\{i\}} K_h + K_d = A - Bp_M + \eta \tag{A1}$$

$$\sum_{h\in\Phi_M} K_h = A_d - B_d p_M + \eta_d \tag{A2}$$

where A, B, A_d and B_d are positive constants and η and η_d are stochastic terms to be determined in equilibrium, and Φ_M is the set of the agents trading in the auction market. Given the assumed difference in risk-attitude, dealer's and investors' conjectures of *residual* demand are to be different. Each investor i calculates the market-clearing price $p_M$by equating total demand to total supply:

$$\sum_{h\in\Phi_M} K_h + K_d = A - Bp_M + \eta + K_i = \sum_{h\in\Phi_M} e_h + (Q_B - Q_A) \tag{A3}$$

Similarly, for the dealer we have:

$$\sum_{h\in\Phi_M} K_h + K_d = A_d - B_d p_M + \eta_d + K_d = \sum_{h\in\Phi_M} e_h + (Q_B - Q_A) \tag{A4}$$

Agents solve their maximization problems given (A3) and (A4): investors maximize (2.1) subject to their wealth constraint (2.2), with $p_J = p_M$ derived from (A3); the dealer solves a similar problem, maximizing (2.3) subject to (2.4) and (A4). From the set of the first-order conditions we obtain:

$$K_i = \left(\frac{R}{B} + b\sigma^2\right)^{-1}\left(\mu - Rp_M + \frac{R}{B}e_i\right) \qquad i \in \Phi_M \tag{A5}$$

$$K_d = \frac{B_d}{R}(\mu - Rp_M) + (Q_B - Q_A) \tag{A6}$$

Aggregating the N_M-1 demand functions (A5) and the dealer's demand function (A6), each investor computes the residual market demand. Likewise the dealer aggregates the N_M investors' demand functions. By replacing into (A1) and (A2) agents are able to determine the coefficients A, B, A_d and B_d, and the disturbances η and η_d:

$$A = \frac{2(N_M-1)\mu}{b\sigma^2}, \quad B = \frac{2(N_M-1)R}{b\sigma^2} \tag{4.1}$$

$$\eta = \frac{1}{2N_M - 1} \sum_{h \in \Phi_M - \{i\}} e_h + (Q_B - Q_A) \tag{A7}$$

$$A_d = \frac{2(N_M - 1)N_M \mu}{(2N_M - 1)b\sigma^2}, \quad B_d = \frac{2(N_M - 1)N_M R}{(2N_M - 1)b\sigma^2}$$

$$\eta_d = \frac{1}{2N_M - 1} \sum_{h \in \Phi_M} e_h \tag{A8}$$

Substituting the values of B and B_d in (A5) and (A6), agents' actual demand functions (3.1) and (3.2) in the text are derived.

Appendix 2.

As mentioned in subsection 3.4, our model admits also non-coexistence equilibria. The equilibrium where only the *dealership* market operates is characterized in the following proposition:

Proposition. There exists a *dealership equilibrium* where agents of both types trade with the dealer ($N_M = 0$ and $N_D = N_\alpha + N_\beta$) and the dealer maximizes his profits. The equilibrium prices in the dealership market are:

$$p_A^{**} = \frac{\mu}{R} + \frac{b\sigma^2}{R} \min\left\{\beta, \frac{1}{2}\left[\nu\alpha + (1-\nu)\beta\right]\right\} \tag{A9}$$

$$p_B^{**} = \frac{\mu}{R} - \frac{b\sigma^2}{R} \min\left\{\beta, \frac{1}{2}\left[\nu\alpha + (1-\nu)\beta\right]\right\} \tag{A10}$$

where $\nu = \frac{N_\alpha}{N_\alpha + N_\beta}$ is the share of agents with large endowment shocks.

Proof. At the prices (p_A^{**}, p_B^{**}), a type-β agent will have no incentive to move to the auction market, being the only investor there, if the following conditions hold:

$$-\beta \,\square\, f\left(p_A^{**}, 1, 0\right) \qquad \text{and} \qquad \beta \geq g\left(p_B^{**}, 1, 0\right) \tag{A11}$$

A fortiori type-α agents prefer the dealership market, since $\alpha > \beta$. When (A11) is not binding the dealer will set the prices derived from the maximization of $E(u_d)$, as defined in (3.13); otherwise, by solving (A11) with the equality signs. ■

If the heterogeneity among investors is not too strong, that is the magnitudes of the endowment shocks are not too different (more precisely, if $\frac{\alpha}{\beta} < \frac{1+\nu}{\nu}$), the dealer sets the prices which maximize (3.13):

$$p_A^{**} = \frac{\mu}{R} + \frac{b\sigma^2}{2R}\left[\nu\alpha + (1-\nu)\beta\right] \tag{A12}$$

$$p_B^{**} = \frac{\mu}{R} - \frac{b\sigma^2}{2R}\left[\nu\alpha + (1-\nu)\beta\right] \tag{A13}$$

On the contrary, if $\frac{\alpha}{\beta} \geq \frac{1+\nu}{\nu}$, the dealer will be constrained by (3.18) and will set the prices needed to capture type-β investors:

$$p_A^{**} = \frac{\mu}{R} + \frac{b\sigma^2}{R}\beta \tag{A14}$$

$$p_B^{**} = \frac{\mu}{R} - \frac{b\sigma^2}{R}\beta \tag{A15}$$

When only the dealer market operates, the size of the spread reflects the degree of heterogeneity of the two types of investors now both trading with the dealer. In fact, with a relatively small difference in the endowment shock (i.e. when $\frac{\alpha}{\beta} < \frac{1+\nu}{\nu}$), the dealer's spread is:

$$S^{**} = \frac{b\sigma^2}{R}\left[\nu\alpha + (1-\nu)\beta\right] \tag{A16}$$

and depends on the relative numerosity of agents of either type, where the term in parentheses defines the average endowment shock of the $N_D = N_A + N_B$ traders. If investors are characterized by widely different endowments (i.e. $\frac{\alpha}{\beta} \geq \frac{1+\nu}{\nu}$), then the spread depends only on the magnitude of the "small" investors' disturbance, β:

$$S^{***} = \frac{2b\sigma^2}{R}\beta \tag{A17}$$