A SWITCH CRITERION FOR DEFINED CONTRIBUTION PENSION SCHEMES

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Abstract

In this paper we try to find an investment strategy which gives a high probability of reaching a given target amount at retirement. This strategy depends on two moments: the first is the appropriate time to stop investing the contributions into equities and the second is the optimal moment to convert the fund of these contributions into bonds (noting that the latter may never occur). We define the strategy and compare it with other investment strategies for Defined Contribution Pension Schemes. Finally we do not only analyse the accumulation phase but also take into consideration what happens during the distribution phase.

Résumé

Dans cette étude, nous allons essayer de définir une stratégie d'investissement qui donne une grande probabilité d'atteindre une somme prédéterminée au moment de la retraite. Cette stratégie dépendra de deux moments: la première est la période appropriée pour investir les apports dans des actions et la seconde est le moment optimal pour convertir les fonds des contributions en titres d'emprunt (en retenant que cette dernière phase peut ne jamais se produire). La stratégie sera définie et comparée aux autres stratégies d'investissement des Schémas de Retraite à Cotisations Définies. En définitive, nous n'analyserons pas seulement la phase d'accumulation mais nous prendrons aussi en compte ce qui se produit lors de la phase de distribution.

Keywords: Defined Contribution Pension Scheme, Accumulation and Distribution Phase, Income Drawdown, Reserve.

1. Introduction♣

In the literature on defined contribution pension schemes there are two different approaches to study the financial risk borne by the member, the theoretical and the empirical approach. The theoretical approach tries to optimise risk and/or return using dynamic programming techniques (among others, Blake et al., 2000a,. Boulier et al., 2000, Deelstra et al., 2000, Haberman &Vigna, 2002, Vigna & Haberman, 2001). The empirical approach examines the currently used investment strategies and tries to propose better alternatives (among others, Blake et al., 2000b, Booth & Yakoubov, 2000, Knox, 1993, Ludvik, 1994). In this paper we adopt the second approach.

A widespread empirical investment policy in defined contribution schemes is the so-called "lifestyle strategy" (among others, Exley et al., 1998, Knox, 1993) whereby the individual gradually converts her/his equity portfolio into a bond portfolio in the last years before retirement. In the first years the investments are made into equities in order to get a high expected yield, and in the last years the investments are made into bonds which have a lower expected yield, but also a lower volatility, in order to prevent unnecessary risk close to retirement.

The idea in this paper is inspired by the fact that, since equities outperform bonds in the long run (see among others, Bodie, 1995), the member of a defined contribution pension scheme needs in general a sufficiently long period of investment in equities before switching into bonds. This period is shorter if returns on equities are high and longer if returns are low. Therefore it depends on equity performance during the accumulation phase and it is not appropriate to determine it in advance using a fixed rule (like in "lifestyle strategy and static portfolio allocations). In this paper we propose a criterion for switching the portfolio from equities into bonds which is partially dynamic, i.e takes into account actual realisations of returns on assets.

We investigate an investment strategy in which the individual makes two different kinds of switches, instead of gradually switching the portfolio in bonds. After investing the contributions in equities for a certain period, the individual will build up a less risky fund with contributions invested in bonds. From this point on (first switch), there are two funds, the first one with the "new" contributions invested into bonds, which will always be invested into bonds, and the second one, with the "old" contributions invested in equities, which will be invested into equities until the switch of the fund occurs (second switch) and into bonds afterwards. The period in which the second switch can take place is from the first switch until retirement but can be extended to the years after retirement as well, if the retiree takes the income drawdown option.

We want to determine the best time for the two different switches trying to maximise the probability of reaching the target fund at retirement, i.e. the best time to start investing the contributions into bonds instead of equities and the best time to switch the equity fund into bonds. In what follows, we propose a method for determining these two moments, investigate and test our strategy by means of Monte Carlo simulations and compare it with other investment strategies.

We will consider not only the pre-retirement period (or accumulation phase), but also the post-retirement period (or decumulation phase), with different assumptions regarding the choice annuitization / drawdown option, depending on whether or not the second switch has occurred

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in the pre-retirement period. Up to our knowledge, it is the first time that the two phases are considered together.

The paper is organised as follows. Section 2 describes the basic strategy. In section 3 we describe the adjustments we make to the basic strategy. Section 4 describes the decumulation phase and section 5 concludes.

2. The basic strategy

2.1 Set up and assumptions

In order to describe the investment strategy that we want to investigate, we determine 6 important moments in the life of the individual:

I The moment when the individual joins the scheme, from this point on all the contributions are invested in equities.

SC Switch of the contributions. The moment when the individual stops investing contributions into equities and starts investing them into bonds. From this point on there are two funds: the equity fund (f_t^{CE}) which consists of all the contributions up to time **SC** invested in equities and the bond fund (f_t^{CB}) which consists of all the contributions from time **SC** on invested into bonds.

R The time of retirement.

A The time when annuitization is compulsory, in case the individual takes the Income Drawdown option (for example when the individual reaches the age of 75, which we will discuss later).

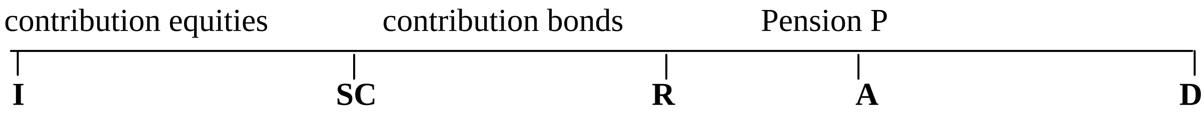
SF Switch of the equity fund. The time at which the equity fund (f_t^{CE}) is switched into bonds. From this point on both portfolios are invested into bonds. This switch-moment will depend on a “switch-criterion” which will be specified later. We assume that if this switch occurs before retirement, then the individual buys a fixed real annuity at time **R** (in this case moment **A** does not play any role).

If this switch has not occurred yet at retirement, the individual will take the income drawdown[1] option, so the switch may occur at any time between **R** and **A**. In this case when the switch criterion is satisfied, the individual instead of converting the equity fund into bonds, buys a fixed real annuity with both funds. If the switch has not occurred yet at time A, then the remaining funds in equities and bonds are used to buy a fixed real annuity.

D The time of death of the individual.

Timeline

[1] The fund remains invested after retirement with a flexible income generated by withdrawals from the fund. Income allowed is broadly between 35% (minimum income that must be drawn) and 100% of the equivalent guaranteed level single life annuity.



We take the initial time **I** to be 0 and the retirement time **R** to be 40 (if the investment strategy does not work at a long horizon, looking at a shorter horizon has even less use), the contribution (c) is a fixed percentage of the real salary, and the real salary remains constant over time.

Assumptions on the assets

Asset returns are assumed to be lognormally distributed. In particular, the return on the bond is e^{μ_t}, with $\mu_t \sim N(\mu, \sigma_\mu^2)$ and the return on equity e^{λ_t}, with $\lambda_t \sim N(\lambda, \sigma_\lambda^2)$. λ_t and μ_t are assumed to be uncorrelated.

We now define a target real return (**r***), which will be used later in determining **SC** and in defining the switch criterion. As the target return we have chosen the Chisini average of μ_t and λ_t relative to the expected return over one year of a portfolio invested equally in the two assets. Therefore **r*** is the solution of $E(e^{r^*}) = E(e^{\frac{\mu_t+\lambda_t}{2}})$ and this means that $r^* = \frac{1}{2} \cdot (\mu + \lambda) + \frac{1}{8} \cdot (\sigma_\lambda^2 + \sigma_\mu^2)$, since λ_t and μ_t are uncorrelated.

2.2 The period in which the contributions are invested into equities

In order to estimate the period in which the contributions should be invested into equities, the first thing an individual has to consider at starting time I, is either her/his required average real return on investments or his/her Target Fund (F_R^{TAR}) at retirement.

This required return depends on the risk aversion of the individual and is supposed to be between the expected return on bonds $(e^{\mu+\frac{1}{2}\sigma_\mu^2})$ and the expected return on equities $(e^{\lambda+\frac{1}{2}\sigma_\lambda^2})$. With this required return (e^{r^*}) we can now calculate the switch year SC of the contributions. We define SC as the solution of the following equation:

$$F_I^{TAR} = F_I^{CE} + F_I^{CB}, \qquad [1]$$

where:

F_I^{TAR} is the fund at time R obtained by investing all the contributions at the rate r*. We call it the initial projected Target Fund at retirement;

F_I^{CE} is the fund at time R obtained by investing a rent of SC contributions in an asset whose return is $E(e^{\lambda_t})$ and then investing this fund for (R-SC)-years in an asset whose real return is $E(e^{\mu_t})$. We call it the initial Projected Fund of contributions invested in equities;

F_I^{CB} is the fund at time R obtained by investing a rent of (R-SC) contributions in an asset whose return is $E(e^{\mu_t})$. We call it the initial projected Fund of contributions invested in bonds.

Or

$$\sum_{j=0}^{(R-1)} c \cdot (e^{r^*})^{(R-j)} = (\sum_{i=0}^{SC-1} c \cdot (e^{\lambda + 1/2\sigma_\lambda^2})^{(SC-i)}) \cdot (e^{\mu + 1/2\sigma_\mu^2})^{(R-SC)} + \sum_{i=SC}^{R-1} c \cdot (e^{\mu + 1/2\sigma_\mu^2})^{(R-i)} ,$$

[2]

which is equivalent to:

$$\sum_{j=0}^{(R-1)} c \cdot (e^{r^*})^{(R-j)} = (\sum_{i=0}^{SC-1} c \cdot (E(e^{\lambda_i}))^{(SC-i)}) \cdot (E(e^{\mu_t}))^{(R-SC)} + \sum_{i=SC}^{R-1} c \cdot (E(e^{\mu_t}))^{(R-i)} .$$

[3]

In other words, SC is the point in time such that investing contributions for the whole period (from I to R) at the required return e^{r^*} is equivalent to investing contributions from I to SC in an asset with constant return equal to $E(e^{\lambda_t})$ (expected return on equities) and then investing the fund and the contributions from SC to R in an asset with constant return equal to $E(e^{\mu_t})$ (expected return on bonds).

We note that SC is deterministic: once the parameters of the asset returns are given, SC is calculated in a deterministic way, using expected values.

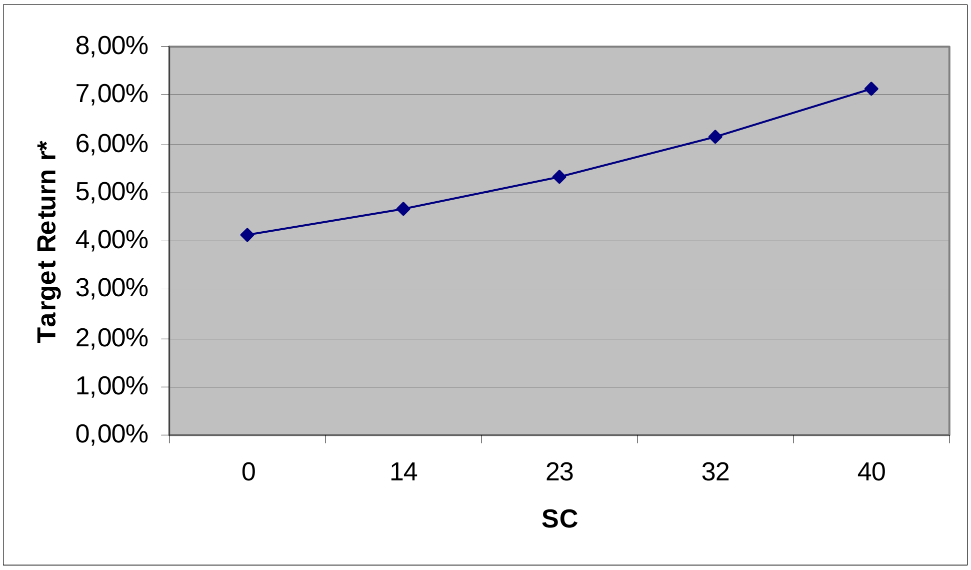
Example:

Let $\mu_t \sim N\left(4\%,(5\%)^2\right)$ and $\lambda_t \sim N(6\%,(15\%)^2)$. Given the parameters of the asset returns, and given that the correlation between equities and bonds is zero, $r^* = 5{,}3125\%$.

With the Target Return (r*) 5,3125%, the year SC in which the contributions are switched from equities to bonds is 23, this means that the first 22 annual contributions are invested in equities and after that the contributions are invested in bonds. Considering different rates for the required real return r*, still bounded between $(\mu + 0{,}5\sigma_\mu^2)$ and $(\lambda + 0{,}5\sigma_\lambda^2)$ (it would be unreasonable to require a return outside this range[2]), we would find the following values for SC as reported in figure 1.

[2] In our example the range is [4,125%; 7,125%], this is [$(\mu + 0{,}5\sigma_\mu^2)$; $(\lambda + 0{,}5\sigma_\lambda^2)$].

Figure 1. The SC for the different target returns r*



Comments

It is clear that if the required return increases the SC increases as well, this means also that the time period in which the SF (switch of the equity fund) can take place (between SC and R) will be shorter. This is consistent with the fact that the individual who requires a high rate of return is less risk averse than the individual who requires a low rate of return, and therefore invests for a longer period into riskier assets. He/she will also have less time before retirement to make the switch of the equity fund.

The projected funds $(F_I^{TAR}, F_I^{CE}, F_I^{CB})$ at time I were calculated with the expected returns on the assets, the actual funds at time t will obviously depend on actual realisations of the asset returns over time.

We consider the following fund values (for $I = 0 \leq t \leq SC$):

$f_t^{CE} = (\sum_{i=0}^{(t-1)} c \cdot \prod_{j=(i+1)}^{t} e^{\lambda_j})$, the value of the fund with the contributions invested into equities at time t;

$f_t^{CB} = 0$, the value of the fund with the contributions invested into bonds at time t (which is zero because we are before time SC);

$f_t^{TOT} = f_t^{CE} + f_t^{CB}$, the total of the two funds built-up;

2.3 The switch criterion for the equity fund

For our investment strategy the switch of the equity fund can occur from time SC on and the switch criterion will be tested yearly from time SC. The switch of the equity fund occurs at time SC if the following holds:

$$\underbrace{\sum_{j=0}^{(R-1)} c\cdot(e^{r^*})^{(R-j)}}_{F_{SC}^{TAR}} \le \underbrace{(\sum_{i=0}^{(SC-1)} c\cdot\prod_{j=i+1}^{SC}(e^{\lambda_j}))\cdot(e^{(\mu+0.5\sigma_\mu^2)})^{(R-SC)}}_{F_{SC}^{CE}} + \underbrace{\sum_{i=SC}^{R-1} c\cdot(e^{\mu+1/2\sigma_\mu^2})^{(R-i)}}_{F_{SC}^{CB}},$$

[4a]

which is equal to:

$$\sum_{j=0}^{(R-1)} c\cdot(e^{r^*})^{(R-j)} \le f_{SC}^{CE}\cdot(e^{(\mu+1/2\sigma_\mu^2)})^{(R-SC)} + \sum_{i=SC}^{R-1} c\cdot(e^{\mu+1/2\sigma_\mu^2})^{(R-i)},$$

[4b]

which is equal to:

$$\sum_{j=0}^{(R-1)} c\cdot(e^{r^*})^{(R-j)} \le f_{SC}^{CE}\cdot(E(e^{\mu_t}))^{(R-SC)} + \sum_{i=SC}^{R-1} c\cdot(E(e^{\mu_i}))^{(R-i)}.$$

[4c]

We notice that at time SC the returns on the contributions invested into equities are known. Therefore in the formula above we use the actual realisations (e^{λ_t}) instead of the expectation (whereas for the return on bonds from SC to R we take the expectation).

$F_{SC}^{CE}, F_{SC}^{TAR}, F_{SC}^{CB}$ are the projected funds at time R calculated at time SC. We observe that F_{SC}^{TAR} and F_{SC}^{CB} are equal to F_I^{TAR} and F_I^{CB}, while F_{SC}^{CE} is different from F_I^{CE} because realized returns are used instead of expectations.

In simpler words, the switch of the equity fund occurs if

$$\sum_{i=0}^{(SC-1)} c\cdot\prod_{j=(i+1)}^{SC} e^{\lambda_j} \ge \sum_{i=0}^{(SC-1)} c\cdot(E(e^{\lambda_t}))^{(SC-i)} \text{ or equivalently } (F_{SC}^{CE} \ge F_I^{CE}),$$

[5]

that is, the switch occurs if the investment returns on equities behaved on average like or better than their expectations.

If this is not the case, and at time SC we have

$$\sum_{i=0}^{(SC-1)} c\cdot\prod_{j=(i+1)}^{SC} e^{\lambda_j} < \sum_{i=0}^{(SC-1)} c\cdot(E(e^{\lambda_i}))^{(SC-i)},$$

[6]

then the switch does not occur and the equity fund will remain invested into equities for the next year. After one year the switch criterion will be tested again and the switch will occur (so SF=SC+1) if the following holds:

$$\sum_{i=0}^{(R-1)} c\cdot(e^{r^*})^{(R-i)} \le (\sum_{i=0}^{(SC-1)} c\cdot\prod_{j=(i+1)}^{SC} e^{\lambda_j})\cdot e^{\lambda_{(SC+1)}}\cdot(E(e^{\mu_t}))^{(R-(SC+1))} + c\cdot e^{\mu_{(SC+1)}}\cdot(E(e^{\mu_t}))^{(R-(SC+1))} + \sum_{i=(SC+1)}^{(R-1)} c\cdot E(e^{\mu_t})^{(R-i)}.$$

[7]

We notice that at time (SC+1) also the return on bonds in year (SC, SC+1) is known ($e^{\mu_{SC}}$), and we use it in the capitalisation of the contribution paid at time SC. The switch criterion will be tested each year up to the first time t at which the criterion is satisfied.

In general, the switch occurs at the first time t when the following holds:

$$F_t^{CE} + F_{SC,t}^{CB} \geq F_I^{TAR} - F_{t,R}^{CB}, \qquad [8]$$

where:

$F_t^{CE} = f_t^{CE} \cdot (e^{\mu + 1/2\sigma_\mu^2})^{(R-t)}$, the Projected Fund of contributions invested into equities for SC<t≤R. With $f_t^{CE} = (\sum_{i=0}^{(SC-1)} c \cdot \prod_{j=(i+1)}^{SC} e^{\lambda_j}) \cdot \prod_{i=SC}^{t} e^{\lambda_i}$, the value of the fund with contributions into equities, if the switch into bonds has not yet taken place;

$F_t^{CB} = F_{SC,t}^{CB} + F_{t,R}^{CB}$, the adjusted projected final value of the portfolio on bonds F_t^{CB} in year SC<t<R;

$F_{SC,t}^{CB} = (\sum_{i=SC}^{t-1} c \cdot \prod_{j=i+1}^{t} (e^{\mu_j})) \cdot (e^{\mu + 1/2\sigma_\mu^2})^{R-t}$, the projected final value of the contributions already invested into bonds from SC up to time t;

F_I^{TAR}, the initial Target Fund at retirement;

$F_{t,R}^{CB} = \sum_{i=t}^{(R-1)} c \cdot (e^{\mu + 1/2\sigma_\mu^2})^{(R-i)}$, the projected value of the future contributions to be invested in bonds.

In other words, the left hand side of the equation is the projected final value of the contributions already invested and the right hand side of the equation is the Final Target Fund minus the projected future contributions.

If the switch has occurred before time t, the value of the equity fund at time t will be:

$$f_t^{CE} = (\sum_{i=0}^{(SC-1)} c \cdot \prod_{j=(i+1)}^{SC} e^{\lambda_j}) \cdot \prod_{i=(SC+1)}^{SF} e^{\lambda_i} \cdot \prod_{i=(SF+1)}^{t} e^{\mu_i}. \qquad [9]$$

The rest of the values remain the same.

If the switch does not take place before retirement the individual chooses the "income drawdown"-option, and the switch criterion after retirement will change. We will discuss this later.

Testing the strategy: a numerical example & simulations

In order to see how the strategy works we carry out some simulations. With $\mu_t \sim N(4\%,(5\%)^2)$, $\lambda_t \sim N(6\%,(15\%)^2)$, $\rho = 0$ and c=1, the Target Fund (F_I^{TAR})

at time R will be *142,50*. If SC = 23, the expected equity amount at retirement (F_I^{CE}) will be *115,94* and the expected bond amount (F_I^{CB}) will be *26,56*.

We make 1.000 Monte Carlo simulations of investment returns of the 2 assets, applying each year the investment strategy previously described. Therefore we introduce the final fund built up at retirement $F_R^{TOT} = F_R^{CE} + F_R^{CB}$, where $F_R^{CE} = f_R^{CE}$ and $F_R^{CB} = \sum_{i=SC}^{R} c \cdot \prod_{j=(i+1)}^{R} e^{\mu_j}$. Then we compare the results we get with the results obtained by investing the whole portfolio into equities for 40 years. From now on we will call the investment strategy based on the switch criterion previously described the "switch strategy".

The results we present are:

- the mean and the standard deviation of the final fund;
- the downside deviation[3] of the final fund and the mean of the shortfall[4] (where by shortfall we mean the difference of the fund from the target, provided that this difference is < 0);
- the probability of failing the target fund;
- the probability of failing the target fund given that the switch of the equity fund occurred before retirement. This probability is very important in this paper because the fund at retirement is lower than the target fund at retirement and is fully invested into bonds. This means no income drawdown, we will specify this later;
- the value at risk at 95% level of the fund and the value at risk at 75% level of the fund.

Table 1. The "40 years 100% equities" versus the "switch strategy (SC=23)"

	40 years 100% equities	Switch strategy (SC=23)
Mean	236,4	158,1
Standard Deviation	202,6	66,7
Downside Deviation	54,3	44,5
Mean shortfall from the F_I^{TAR}	46,3	35,9
$P(F_R^{TOT} < F_R^{TAR})$	39,4%	43,3%
$P(F_R^{TOT} < F_R^{TAR} \mid SF < 41)$	Does not apply	25,5%
VaR 95%	58,3	67,8
VaR 75%	110,1	117,8

Comments

We see that the average value of the final fund is much higher investing 40 years fully in equities than the average final value of the "switch strategy", while the

[3] $= \sqrt{(1/k) \cdot \sum_{j=1}^{k} (F_R^{TOT_j} - F_R^{TAR})^2}$, where $(F_R^{TOT_j} - F_R^{TAR}) < 0$ for j= 1,2,…,k.

[4] $= \frac{1}{k} \cdot \sum_{j=1}^{k} (F_R^{TAR} - F_R^{TOT_j})$, where $(F_R^{TAR} - F_R^{TOT_j}) > 0$ for j= 1,2,…,k.

> "switch strategy" has an even higher probability of failing the target. All the other risk measures indicate that the "switch strategy" is less risky. Finally we notice that the probability of failing the target after the SF is near 25,5%: this means that the switch criterion is not sufficient in more than 1 out of 4 cases, that is very high.

Up to now the switch only makes sure that, from when it is satisfied, the equity risk is eliminated, but the risk of the return on bonds still remains.

3. Adjustments to the basic model

3.1 Yearly targets and a flexible SC

In order to monitor the growth of the fund over time we introduce yearly targets, whose definition is different in the periods before and after SC.

For $0 \le t \le SC$ the yearly target is:

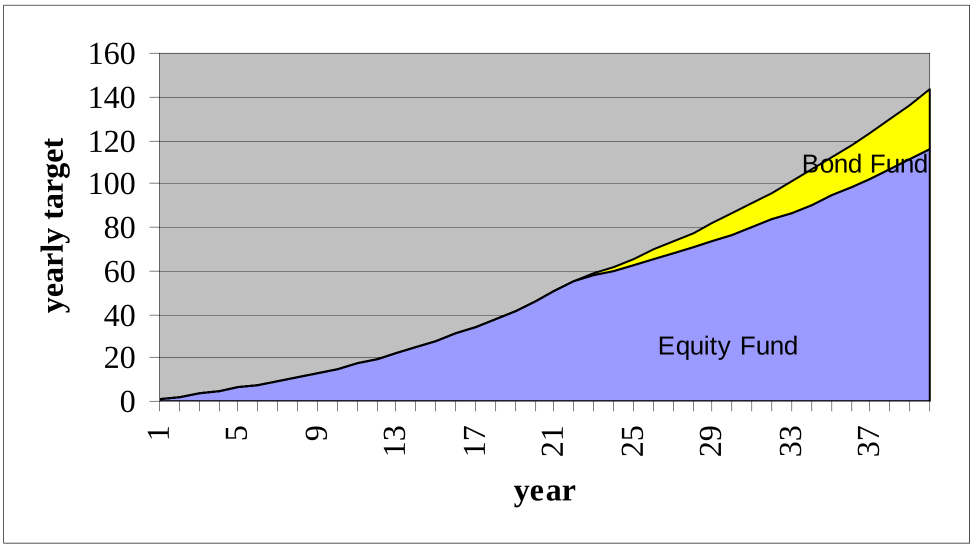
$$YT_t = \sum_{i=0}^{t-1} c \cdot (e^{(\lambda+0.5\sigma_\lambda^2)})^{(t-i)} = \sum_{i=0}^{(t-1)} c \cdot E(e^{\lambda_t})^{(t-i)} \qquad [10]$$

and from SC<t$\le R$ the yearly target is:

$$YT_t = \sum_{i=0}^{(SC-1)} c \cdot (e^{(\lambda+0.5\sigma_\lambda^2)})^{(SC-i)} \cdot (e^{(\mu+0.5\sigma_\mu^2)})^{(t-SC)} + \sum_{i=SC}^{(t-1)} c \cdot (e^{(\mu+0.5\sigma_\mu^2)})^{(t-i)} . \qquad [11]$$

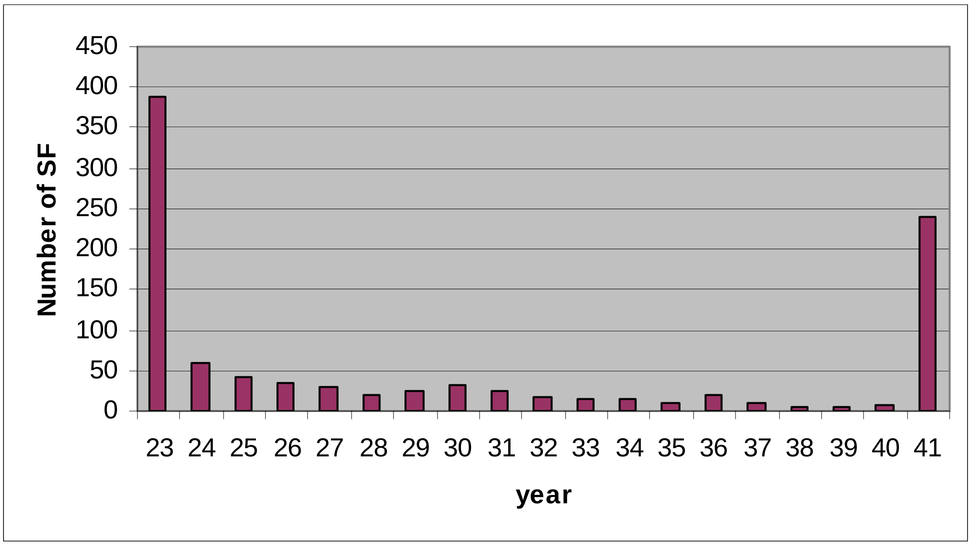
We also need f_t^{TOT}, the total of the two funds built-up at time t, where $f_t^{TOT} = f_t^{CE} + f_t^{CB}$.

Figure 2. The yearly target built up by the two funds



The distribution of SF over the 1.000 simulations is plotted in the figure 3. For the moment we assume that if the switch has not occurred at retirement (SC=41) it will occur the year after (41). Those people would take the income drawdown option which we will consider later.

Figure 3. The Distribution of the SF



If we simulate (1.000 simulations) with the SC=23, the criterion above is satisfied in less then 40% of the cases after 23 years, so P(SF=23)<40%. We think that this result can be explained by the fact that in the lognormal distribution the median (or 50th percentile) is lower than the mean. Therefore we would expect in more than 50% of the cases a return lower than the expected value. This would lead to the criterion not to be satisfied in more than 50% of the cases.

Considering the example introduced before, the expectation of the sum of the premiums invested in equities at the expected switch time (23 year) $YT_{23} = (\sum_{i=0}^{22} c \cdot \prod_{j=(i+1)}^{23} E(e^{\lambda_t}) = 55.1783$.

We want to analyse what happens if we change the SC from 23 to respectively 24, 25,..., 31, that means investing extra years in equities, and look at the probability that the previously defined yearly target fund $(YT_{24},...,YT_{31})$, for which the SC is fixed at year 23, in that year will be reached.

Table 2. Results of investing the contribution extra years in equities

Year t	23	24	25	26	27	28	29	30	31
Average $f_{SC=t}^{TOT}$	55,4	60,6	65,8	71,8	79,2	85,6	92,9	100,3	108,7
YT_{SC} at t	54,9	58,2	61,7	65,3	69,1	73,1	77,2	81,5	86,0
$P(SF = SC \mid SC = t)$	38,8%	41,6%	44,2%	45%	46,7%	47,7%	50,2%	52,9%	54,7%
$P(SF \le t \mid SC = 23)$	38,8%	44,9%	49,0%	52,5%	55,5%	57,5%	59,9%	63,2%	65,6%
$P(F_R^{TOT} < F_R^{TAR} \mid SC = t)$	25,5%	23,7%	21,6%	21,0%	18,6%	18,2%	17,0%	15,9%	14,0%

Comments

If you invest the contributions for some extra years in equities the difference between the total fund at time t and the yearly target at time t is on average increasing. $P(SF = SC \mid SC = t)$ is smaller than $P(SF \le t \mid SC = 23)$ because in the first case the switch criterion is tested only at time SC = t, in the second case the “switch strategy” is tested at any time between SC=23 and time t (it is the accumulation of all the switches from SC=23 until t). The probability of failing the target if the switch is made after investing some extra years in equities is decreasing.

3.2 The need for a buffer or reserve

We now want to give an estimate of the volatility of the final fund (F_R^{TOT}) under the assumption that the yearly target YT_t is exactly reached after respectively 22 and 30 years, and is not reached before, and SC is 23 (this means you invest the contributions into equities for 22 years and convert the whole fund f_t^{CE} from equities into bonds at year 23 and year 31).

Suppose YT_t is exactly sufficient at the beginning of year 23, which means that $F_{23}^{CE} = F_{23}^{TAR} - F_{23}^{CB}$, then the final fund at retirement will be:

$$(YT_{23} \cdot \prod_{i=23}^{40} e^{\mu_i} + \sum_{i=23}^{39} c \cdot \prod_{j=(i+1)}^{40} e^{\mu_j}).$$

[12]

We carry out again 1.000 simulations for investment returns and we look at the probability of failing the target, the mean shortfall and the downside deviation of the final fund. The probability of failing the target is 53,4% with a mean shortfall of the final fund at R of 20,4 and a downside deviation of the final fund of 24,5.

If YT_t is exactly sufficient at year 31, the final fund at retirement will be:

$$(YT_{31} \cdot \prod_{t=31}^{40} e^{\mu_t} + \sum_{t=30}^{39} c \cdot \prod_{j=(t+1)}^{40} e^{\mu_j}).$$

[13]

We use the same 1.000 simulations as before (considering the investment returns simulated in the last ten years). The probability of failing the final target is 52,3% with a mean shortfall at retirement (R) of 16,3 and a downside deviation of the final fund of 19,6.

Table 3 shows results when YT_t is exactly reached after 23, 25,…, 39 years.

We define:

SMS= mean shortfall from F_I^{TAR}, we call it “simulated mean shortfall”.

SDD= Downside deviation of the F_I^{TAR}, we call it “simulated downside deviation”.

Table 3. the results when $f_t^{TOT} = YT_t$ at year 23, 25,..., 39

SF	23	25	27	29	31	33	35	37	39
SMS	20,4	19,1	17,9	16,8	16,3	14,1	12,5	9,5	5,7
SDD	24,5	23,0	21,7	20,4	19,6	17,1	15,3	11,7	7,1
$P(F_R^{TOT} < F_R^{TAR} \mid f_t^{TOT} = YT_t)$	53,4%	52,2%	52,7%	51,9%	52,3%	51,8%	50,7%	52,4%	50,7%

We observe two things, the mean shortfall and the downside deviation of the final fund are decreasing as the SF is increasing and in both cases the probability of failing the target with the criterion we use is very high. This is a reasonable result because, when SF is high, retirement is closer and risk on bond return affects to a lesser extent the final fund result.

Given this high probability of failing the final target, we think that the yearly target at SF is not "enough", it does not include a reserve or buffer in order to hedge against poor performance of bonds.

How can a reserve be created? The criterion has to be changed. An easy way of increasing the expectation of the value of the fund at retirement is simply to invest the contributions some extra years in equities, as we saw before. We assume, for example, that SC=31 (which means investing all the annual contributions the first 30 years in equities, without a buffer), carry out 1.000 simulations and compare it with the "lifestyle strategy"[5] of annually switching 10 % from equities into bonds every year during the last ten years before retirement and with the "switch strategy SC=23".

Table 4. The "lifestyle strategy" versus the "switch strategy" for both SC=23 and SC=31

	Lifestyle strategy	Switch strategy (SC=23)	SC=31
Mean	201,8	158,1	184,8
Standard deviation	136,4	66,7	104,7
Downside deviation	48,7	44,5	52,0
Mean shortfall from F_I^{TAR}	42,0	35,9	42,7
P($F_R^{TOT} < F_R^{TAR}$)	40,5%	43,3%	36,0%
$P\left(F_R^{TOT} < F_R^{TAR} \mid SF < 41\right)$	Does not apply	25,5%	14,0%
Value at Risk 95%	69,4	67,8	60,9
Value at Risk 75%	111,6	119,6	123,7

Comments

Changing the SC from 23 to 31 (without including a reserve) gives a higher average final fund while the two probabilities of failing the target decrease substantially. The other risk measures of the "switch strategy SC=23" are lower, but if we take the 360 lowest values of the "switch strategy (SC=23)"(considering that the probability of failing the target in SC=31 is 36%), the downside deviation of the final fund is 49,1 and the mean shortfall is 42,5 , not much better than in the "SC=31 strategy" (underlining the fact that sometimes you must be careful in drawing conclusions when comparing the same risk measure on different strategies). The "lifestyle strategy" has a higher average than the two switch strategies, the probability of failing the target is between the two. Finally, if we look at the distribution of the fund we see that in the extreme cases the "value at risk 95%" the "lifestyle strategy" has a higher value than the "switch strategies" while a lower value regarding the "value at risk 75%".

Considering the possibility of building up a reserve, one should estimate how much the individual needs as a reserve at each time t. To do this, we look at the downside deviation or the mean shortfall of the final fund. We have seen before both that the downside deviation and the mean shortfall are decreasing as the SF increases.

[5] The value of the fund of the "lifestyle strategy" at time t , (f_t^{LS}), is:

$f_t^{LS} = (f_{(t-1)}^{LS} + c) \cdot (1 + \omega_{EQ} \cdot e^{\lambda_t} + (1 - \omega_{EQ}) \cdot e^{\mu_t})$,with ω_{EQ} = the proportion of fund invested in equities at the beginning of year t.

There are many ways of defining a switch criterion which allows for the presence of a reserve. In general, the switch of the Equity Fund at time t will occur if:

$$(F_t^{CE} + F_{SC,t}^{CB}) \geq (1 + Buffer_t) \cdot (F_R^{TAR} - F_{t,R}^{CB}) \; .$$

[14]

For the $Buffer_t$ we can use for example:

1. $buffer_t = \dfrac{SMS_t}{YT_t}$

[15a]

or

2. $buffer_t = \dfrac{SMS_t}{YT_t \cdot (e^{(\mu + 0.5\sigma_\mu^2)})^{(R-t)}} \; .$

[15b]

The first definition takes as the buffer the full estimated shortfall at time t divided by the yearly target at time t, while the second takes the discounted estimated shortfall at time t divided by the yearly target at time t. We will consider the first definition.

The buffer we define depends on the simulations on investments we made. To make it more general, we make a linear regression[6] to have smoothed results.

Table 5. "Switch strategy (SC=23)" versus "switch strategy (SC=23) with buffer"

Switch strategy	SC=23	SC=23 with buffer
Mean	158,1	167,8
Standard deviation	66,7	70,8
Downside deviation	44,5	52,5
Mean shortfall from the F_I^{TAR}	67,8	45,8
P($F_R^{TOT} < F_R^{TAR}$)	43,5%	34,7%
$P\left(F_R^{TOT} < F_R^{TAR} \mid SF < 41\right)$	25,5%	5,2%
Value at Risk 95%	67,8	66,4
Value at Risk 75%	117,8	114,2

Comments

The strategy with buffer has a higher mean and a lower probability of failing the target. If we look at the 95% and the 75% value at risk we see that the values of the switch strategy without buffer are only slightly higher. The strategy with buffer seems to be more appropriate.

[6] See appendix for the linear regression we used.

Table 6. "Switch strategy (SC=31)" versus "switch strategy (SC=31) with buffer"

Switch strategy	SC=31	SC=31 with buffer
Mean	184,8	187,6
Standard deviation	104,7	105,1
Downside deviation	42,2	56,4
Mean shortfall from the F_I^{TAR}	42,9	49,0
P($F_R^{TOT} < F_R^{TAR}$)	36,0%	32,7%
$P\left(F_R^{TOT} < F_R^{TAR} \mid SF < 41\right)$	14%	4,0%
Value at Risk 95%	60,9	60,9
Value at Risk 75%	122,6	117,7

Comments

The strategy with buffer has a slightly higher mean, but the two probabilities of failing the target are significantly lower while the values at risk are not much lower.

We have also looked at an alternative strategy: SC is variable, with a minimum of 23 and a maximum of 31 years, and the SC now occurs when the switch criterion on the equity fund holds or after 30 years (if the criterion is not satisfied between 23 and 31). The maximum of SC has been chosen to be 31 because in the "lifestyle strategy" after 30 years the individual starts investing into bonds.

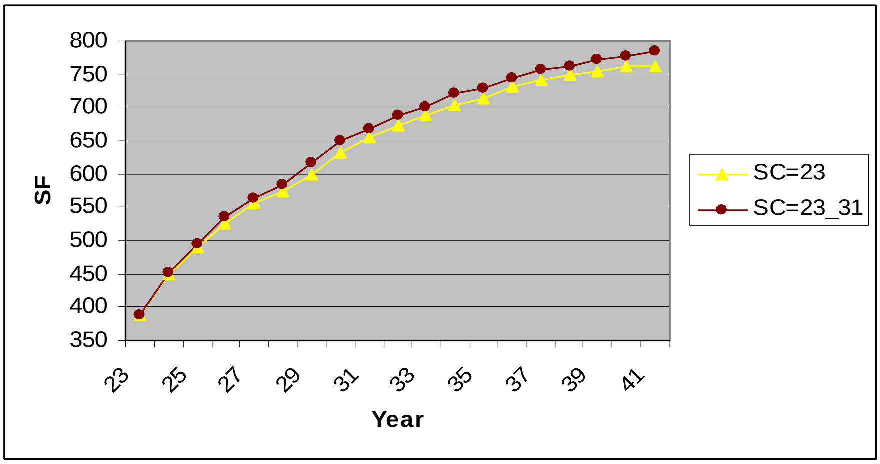
For this strategy the switch occurs at time t, $23 \le t \le 31$, if the following holds:

$$\sum_{i=0}^{(R-1)} c \cdot (e^{r^*})^{(R-i)} \le (\sum_{i=0}^{(t-1)} c \cdot \prod_{j=(i+1)}^{t} e^{\lambda_j}) \cdot (E(e^{\mu_t}))^{(R-t)} + c \cdot e^{\mu_t} \cdot (E(e^{\mu_t}))^{(R-t)} + \sum_{i=t}^{(R-1)} c \cdot E(e^{\mu_t})^{(R-i)} .$$

[16]

Adopting this alternative strategy the cumulative distribution of SF compared to the "switch strategy (SC=23)" is as displayed in Figure 4.

Figure 4. the cumulative distribution of SF



The number of SF at year 23 is the same, but afterwards the number of SF at any year $24 \le t \le 40$ with the variable SC is higher. Table 7 gives the results of the simulations with the flexible SC.

Table 7. “Switch strategy (SC=23_31)” versus “switch strategy (SC=23_31) with buffer”

Switch strategy	SC=23_31	SC=23_31 with buffer
Mean	159,4	169,2
Standard deviation	66,9	71,4
Downside deviation	46,5	56,8
Mean shortfall from the F_I^{TAR}	36,8	48,9
P($F_R^{TOT} < F_R^{TAR}$)	41,4%	32,1%
$P\left(F_R^{TOT} < F_R^{TAR} \mid SF < 41\right)$	25,4%	5,8%
Value at Risk 95%	63,46	60,9
Value at Risk 75%	120,65	121,2

Comments

Also in this case the strategy with buffer seems better and it has the lowest probability of failing the target at retirement of all the strategies we tested.

So we conclude that in general the switch strategies with a buffer seem to be better than the strategies without a buffer. What about the “switch strategy” compared to the “lifestyle strategy” and the strategy of investing fully into equities? To do this we take the “switch strategy (SC=31) with buffer” because it has the highest average final fund of all the switch strategies (but still lower than the “lifestyle strategy” and the “full equity strategy”).

Table 8. The “lifestyle strategy” versus “switch strategy (SC=31) with buffer”

	Lifestyle strategy	Switch SC=31 with buffer
Mean	201,8	187,6
Standard deviation	136,4	105,1
Downside deviation	48,7	56,4
Mean shortfall from the F_I^{TAR}	42,0	49,0
P($F_R^{TOT} < F_R^{TAR}$)	40,5%	32,7%
$P\left(F_R^{TOT} < F_R^{TAR} \mid SF < 41\right)$	Does not apply	4,0%
Value at Risk 95%	69,4	60,9
Value at Risk 75%	111,6	117,7

Comments

The mean of the “lifestyle strategy” is much higher, but the probability of failing the target as well. The difference between the value at risk 95% indicates that in extreme cases the “SC=31” is much lower.

Table 9. "40 years 100% equities" versus the "switch strategy (SC=31) with buffer":

	40 years 100% equities	Switch SC=31 with buffer
Mean	236,4	187,6
Standard deviation	202,6	105,1
Downside deviation	54,3	56,4
Mean shortfall from the F_I^{TAR}	46,3	49,0
P($F_R^{TOT} < F_R^{TAR}$)	39,4%	32,7%
$P\left(F_R^{TOT} < F_R^{TAR} \mid SF < 41\right)$	Does not apply	4,0%
Value at Risk 95%	58,3	60,9
Value at Risk 75%	110,1	117,7

Comments

Here the mean of the "40 years 100% in equities" is much higher than that of the "switch (SC=31) with buffer", but it is also more risky, which seems quite logical.

4. The decumulation phase

4.1 Retirement and beyond

At this stage the individual has retired and thus needs a pension income. There is the choice between income drawdown and a fixed real annuity[7]. The assumption in this paper is that, if the switch has occurred before retirement, a fixed real annuity is bought at retirement and if this is not the case the individual chooses "income drawdown without bequest"[8] until the switch criterion is satisfied.

The annuity option

In case the annuity option is chosen, the SF has occurred before R and at R the annuity will be bought. The yearly annuity pension $(P_{\ddot{a}})$ will be the total fund at time R, divided by the annuity factor[9]: $P_{\ddot{a}} = (\frac{F_R^{TOT}}{\ddot{a}_{x+R}})$, where x is the age of the individual when joining the scheme. This yearly pension will be greater than the target pension if $F_R^{TOT} \geq F_R^{TAR}$, but will be lower if $F_R^{TOT} \leq F_R^{TAR}$.

At R this fund is 100% invested in bonds, like at the end of the "lifestyle strategy". The difference between the "switch strategy" and the "lifestyle strategy" is that with the "switch strategy" the portfolio invested in bonds is used at R to buy annuities, whereas with the "lifestyle strategy", if the income drawdown option is taken, the individual has to convert again the portfolio from bonds into equities. In fact, during the income drawdown it seems to be optimal (among others, Blake et al., 2001, Gerrard *et al.*,2003, Kapur & Orszag, 1999,

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[7] Mortality table used: ISTAT, "Annuario statistico italiano 2000, tavole di mortalità per sesso e per éta - 1996".

[8] This means that should the individual die, the money goes to other pensioners of the scheme and not to relatives.

[9] The rate of interest applied to the fixed real annuity $\ddot{a}_{x+R}$ will be taken fixed, v= $e^{-\mu+0.5\sigma^2}$, with the same expected rate of return as the investment into bonds. This means that we do not take into account the "annuity risk", taking $\ddot{a}_{x+R} = \sum_{i=0}^{\infty} e^{-\mu+0.5\sigma^2} \cdot {}_i p_{x+R}$ and initial age x = 25 years and R=40.

Yaari, 1965) investing the portfolio at least partially into equities[10]. Investing fully in bonds during income drawdown is not suitable because in absence of bequest motives annuities would perform better than 100% bonds, because of mortality drag, hedge against longevity risk, absence of volatility etc etc. Thus, the choice of the income drawdown option at retirement in a scheme where the lifestyle strategy has been applied produces a discontinuity in the portfolio composition.

Income drawdown

We assume that for the "income drawdown option" the pension P withdrawn each year is $P = \frac{F_R^{TAR}}{\ddot{a}_{x+R}}$. This is the Target Fund divided by the factor of the fixed real annuity at retirement, we take this pension because we want to analyse the strategy, which gives the highest probability of reaching F_t^{TAR}. In real life there are restrictions on the amount of pension to be withdrawn from the fund, the pension that can be taken in the scenarios in which we decide to take income drawdown will always be less then P (because the actual fund is lower than the F_R^{TAR}, otherwise the annuity would be bought, see later).

The targets F_t^{TAR} in the years after retirement will change to $P \cdot \ddot{a}_{x+t}$ (t>R). At age 75 the "drawdown option" is no longer allowed, with the remaining fund an annuity will be bought. In our example the individual needs a fund of $P \cdot \ddot{a}_{75} = 46{,}45$ at age 75. If the "income drawdown" option is taken then the pension P will be deducted from the fund in bonds and if this fund is not sufficient pension will be deducted from the fund in equities. The formula for the "switch criterion" after retirement now becomes:

$$\underbrace{((f_t^{CE}\ ?\ \min(f_t^{CB}\ ?\ P;0))\ ?e^{?_t}\ ?\ \max(f_t^{CB}\ ?\ P;0)\ ?e^{?_t})\ ?(1\ ?\ \frac{q_t}{p_t})}_{F_t^{TOT}}\ ?\ \underbrace{P\ ?\ddot{a}_{x?t}}_{F_t^{TAR}} \qquad [17]$$

where $(1+\frac{q_t}{p_t})$ is the bonus factor for pooling (like in Blake et al.,2001).6

This means that the income drawdown first will be taken from the fund invested in bonds while it is sufficient (this will happen with high probability within 2 or 3 years after retirement) and then money will be withdrawn from the fund in equities.

We compare the investment strategies of the accumulation phase and analyse the effect of the possibility to do income drawdown. As the starting point for each strategy we do not make 1.000 simulations, but we go on with all the simulations in which the fund did not reach the target at R (which is the probability of failing the target at retirement); this makes it a bit more complicated to compare the strategies.

[10] Khorasanee (1996) observes "An income withdrawal fund must earn a higher return than a whole life annuity fund... an income withdrawal fund is likely to be invested wholly or partly in equities, as equities are expected to outperform the government bonds held by insurer's annuity funds".

The results we present in the tables are:

Above the thick line:

- the probability of failing the Target Fund (142,51) at retirement;
- the conditional probability of failing the target at retirement given that the switch of the equity fund has occurred;
- the probability of not taking the income drawdown option while the Target Fund at retirement has not been reached. The SF instead has occurred;
- the expectation of the final fund given that the SF has occurred and final target has not been reached.

Under the thick line (analyses of the drawdown):

- the probability of starting the income drawdown, the Target Fund has never been reached;
- the average of the total fund at the start of the income drawdown;
- the probability of reaching the target after retirement;
- the average number of years after retirement at which the target has been reached, given that it has been reached;
- the probability of not going bankrupt before R+10 but also not being able to make the switch;
- the average remaining fund at R+10 (recalling that at R+10 the F_{R+10}^{TAR} = 46,45);
- the probability of going bankrupt before time R+10;
- the average year in which bankruptcy occurs.

Table 10. "Switch strategy (SC=23)" versus "switch strategy (SC=23) with buffer"

Switch strategy	SC=23	SC=23 with buffer
$P(F_R^{TOT} < F_R^{TAR})$	43,2%	34,7%
$P(F_R^{TOT} < F_R^{TAR} \mid SF < 41)$	25,5%	5,2%
P(No Drawdown & $F_R^{TOT} < F_R^{TAR}$)	19,4%	3,6%
$E(F_R^{TOT} \mid NoDrawdown \,\&\, F_R^{TOT} < F_R^{TAR})$	126,4	130,6
P(Drawdown & $F_R^{TOT} < F_R^{TAR}$)	23,8%	31,1%
$E(F_R^{TOT} \mid Drawdown)$	89,9	92,8
P(Switch between R and (R+10))	6,2%	9,4%
Average year switch after R	4,2	4,1
$P(0 < F_{R+10}^{TOT} < F_{R+10}^{TAR} \,\&\, F_t^{TOT} \geq 0$;R<t<(R+10))	4,7%	5,6%
$E(F_{R+10}^{TOT} \mid F_{R+10}^{TOT} < F_{R+10}^{TAR})$	19,9	20,6
$P(Ruin)$	12,9%	16,1%
Average year of Ruin after R	7,9	8,0

Comments

Summing the probability of drawdown and the probability of no drawdown, you get the probability of failing the target at retirement.

No income drawdown

The average total fund of the individuals who do not take the “income drawdown option” is quite high (126,4 and 130,6) with respect to the Target Fund of 142,51. This is reasonable because the fund was higher than the yearly target at least once (so the switch was made), but then went below the target; in the case of income drawdown the yearly target has never been reached so the average will be lower. We see that the probability of not reaching the Target Fund, given the SF has occurred, is much higher in the “switch strategy” without a buffer (25,5% versus 5,2%); we think this is another strong reason to include a buffer.

Income Drawdown

Summing the three percentages of $P(F_{R+10}^{TOT} < F_{R+10}^{TAR})$, $P(Ruin)$ and P(Switch between R and R+10) gives the P(Drawdown & $F_R^{TOT} < F_R^{TAR}$). Thus for the “SC=23 with buffer” we have: 5,6%+9,4%+16,1% = 31,1%. The total probability of not reaching the target for the strategy “SC=23 with buffer” is 25,3% and is the sum of:

- 3,6%, the probability of not taking the income drawdown, the individual gets a pension, but not as high as wanted;
- 5,6%, the probability that at time R+10 the total fund left is not sufficient to buy the annuity desired;
- 16,1%, the probability of going bankrupt before time R+10 (we assume that the individual remains alive during the income drawdown).

Table 11. “Switch strategy (SC=23_31)” versus “switch strategy (SC=23_31) with buffer”

Switch strategy	SC=23_31	SC=23_31 with buffer
$P(F_R^{TOT} < F_R^{TAR})$	41,4%	32,0%
$P(F_R^{TOT} < F_R^{TAR} \mid SF < 41)$	25,3%	5,7%
P(No Drawdown & $F_R^{TOT} < F_R^{TAR}$)	19,8%	4,1%
$E(F_R^{TOT} \mid NoDrawdown \,\&\, F_R^{TOT} < F_R^{TAR})$	126,2	131,3
P(Drawdown & $F_R^{TOT} < F_R^{TAR}$)	21,6%	27,9%
$E(F_R^{TOT} \mid Drawdown)$	87,0	88,4
P(Switch between R and (R+10))	5,7%	7,5%
Average year switch after R	3,8	3,9
$P(0 < F_{R+10}^{TOT} < F_{R+10}^{TAR} \,\&\, F_t^{TOT} \geq 0$;R<t<(R+10))	3,4%	4,4%
$E(F_{R+10}^{TOT} \mid F_{R+10}^{TOT} < F_{R+10}^{TAR})$	18,7	20,2
$P(Ruin)$	12,5%	16,0%
Average year of ruin after R	7,6	7,6

Comments

In general the results are very similar to the strategies with SC=23. The total probability of not reaching the target decreases in comparison with the strategies with SC=23, but on average ruin occurs earlier and the average fund after 10 years is a bit smaller.

Table 12. "Switch strategy (SC=31)" versus "switch strategy (SC=31) with buffer"

Switch strategy	SC=31	SC=31 with buffer
$P(F_R^{TOT} < F_R^{TAR})$	36,0%	32,7%
$P(F_R^{TOT} < F_R^{TAR} \mid SF < 41)$	14,0%	4,0%
P(No Drawdown & $F_R^{TOT} < F_R^{TAR}$)	10,4%	2,8%
$E(F_R^{TOT} \mid NoDrawdown \,\&\, F_R^{TOT} < F_R^{TAR})$	130,0	133,9
P(Drawdown & $F_R^{TOT} < F_R^{TAR}$)	25,6%	29,9%
$E(F_R^{TOT} \mid Drawdown)$	87,4	89,7
P(Switch between R and (R+10))	6,9%	8,7%
Average year switch after R	3,9	3,0
$P(0 < F_{R+10}^{TOT} < F_{R+10}^{TAR} \,\&\, F_t^{TOT} \geq 0$;R<t<(R+10))	4,2%	4,9%
$E(F_{R+10}^{TOT} \mid F_{R+10}^{TOT} < F_{R+10}^{TAR})$	19,8	19,9
$P(Ruin)$	14,5%	16,3%
Average year of ruin after R	7,6	7,7

Comments

The results are again quite similar to the other switch strategies.

We finally compare the "SC=31 with buffer" with "40 years 100% into equities".

Table 13. "40 years 100% equities"[11] versus "switch strategy (SC=31) with buffer"

	40 years 100% equities	Switch SC=31 with buffer
$P(F_R^{TOT} < F_R^{TAR})$	39,4%	32,7%
$P(F_R^{TOT} < F_R^{TAR} \mid SF < 41)$	Does not apply	4,0%
P(No Drawdown & $F_R^{TOT} < F_R^{TAR}$)	Does not apply	2,8%
$E(F_R^{TOT} \mid NoDrawdown \,\&\, F_R^{TOT} < F_R^{TAR})$	Does not apply	133,9
P(Drawdown & $F_R^{TOT} < F_R^{TAR}$)	39,4%	29,9%

[11] In the strategy "40 years 100% equities" the switch from equities to bonds never occurs, so there is no SF and the option of No Drawdown does not occur. After R the fund of this strategy will remain invested fully into equities.

$E(F_R^{TOT} \mid Drawdown)$	96,2	87,4
P(Switch between R and (R+10))	16%	8,7%
Average year switch after R	3,6	3,0
$P(0 < F_{R+10}^{TOT} < F_{R+10}^{TAR} \,\&\, F_t^{TOT} \geq 0$;R<t<(R+10))	6,3%	4,9%
$E(F_{R+10}^{TOT} \mid F_{R+10}^{TOT} < F_{R+10}^{TAR})$	18,1	19,9
$P(Ruin)$	17,1%	16,3%
Average year of ruin after R	7,5	7,7

Comments

The total probability of failing the target for the strategy “40 years 100% equities” is 17,1%+6,3%=23,4%, while the total probability of failing the target of the “switch strategy (SC=31) with buffer” is 16,3%+4,9%+2,8%=24,0%. This means that the strategy “40 years 100% equities”, which is considered very risky in the accumulation phase, actually has the highest probability of reaching the target if income drawdown is considered as well. Of the individuals not reaching the target at R (39,4%), many (16%) will reach the target before R+10. The “switch strategy (SC=31) with buffer” has the advantage that on average the switch is made earlier and the 2,8% which does not take the income drawdown, without reaching the target, still have on average of 133,9 to buy an annuity, that means more than 90% of the target.

5. Conclusion and further research

In this paper the idea has been to try to find a suitable strategy in order to reach the Target Fund at retirement. This is done by taking all the equity risk at the lower ages, as is common in Defined Contribution Schemes, investing the contributions into bonds from a certain moment onwards and waiting for the right moment for switching the equity fund from equities into bonds (buying bonds with the contributions in the meantime). It seems also important to consider both the periods before and after retirement, since we have the “income drawdown” opportunity. Investing fully in equities seems to be less risky than usually considered and the “lifestyle strategy” less appropriate.

Analysing weak and strong points of this strategy, we think that the main weaknesses are:

1. optimality. A critic that can be moved to the “switch strategy” is that it is not “optimal” in the sense of the dynamic programming approach; however, also the “lifestyle strategy”, as well as many other investment strategies proposed in the actuarial literature are not optimal in that sense;
2. mean. The best strategy that we found, the “SC=31 with buffer”, produces an average final fund that is still lower than the average final fund produced by the “lifestyle strategy” and the “40 years 100% equities strategy”;
3. length of the strategy. The strategy is appropriate for long periods only. However, most of the people work for a long period before retirement and, in the case of DC schemes, the position of a member can be moved from one scheme to another in case of turnover, without losing value;
4. annuity risk. The value of the annuity at retirement and after is taken to be constant, whereas it is not, depending on the yields on bonds at the time of purchase. Thus, the annuity risk is not taken into consideration;

5. salary. The salary is taken to be constant over time. Should the salary increase over time, the estimated SC and SF would be higher, implying riskier strategies;
6. no correlation. Assets are assumed to be uncorrelated, which is not realistic.
7. mortality. Mortality is not taken into account in the decumulation phase.

We think that the interesting points are the following:

1. the switch criterion is dynamic, it evolves over time taking into account actual realisations of returns on assets and bonds (i.e. the past history);
2. we consider both the accumulation and the decumulation phase together. In the literature, the two phases are usually considered separately. A dynamic programming approach that considers the two phases would complicate the tractability of the model and this is probably the reason why this has not been done yet (up to our knowledge). This could be an interesting subject for further research;
3. the idea of splitting up the two moments of switch of the contributions from equities into bonds and the switch of the equity fund from equities into bonds is new;
4. the switch criterion is linked to the achievement of a certain target: the switch from equities into bonds occurs when and if there are good chances of reaching the target fund at retirement;
5. in the paper an indication is given for the reserve needed at each year t, in order to compensate for the future risk on bonds.

Finally we suggest three elements for further research that might improve the investment strategy:

1. finding a more appropriate estimate for the buffer needed in each year;
2. changing the switch criterion in such a way that it takes into account the yield given on bonds at the moment of switching from equities into bonds;
3. adding the option of switching from equities not only into bonds, but also into deferred annuities. We think this might improve the strategy, give higher results and lead to lower variance of the results, with the disadvantage that buying deferred annuities implies no bequest in case of death before retirement.

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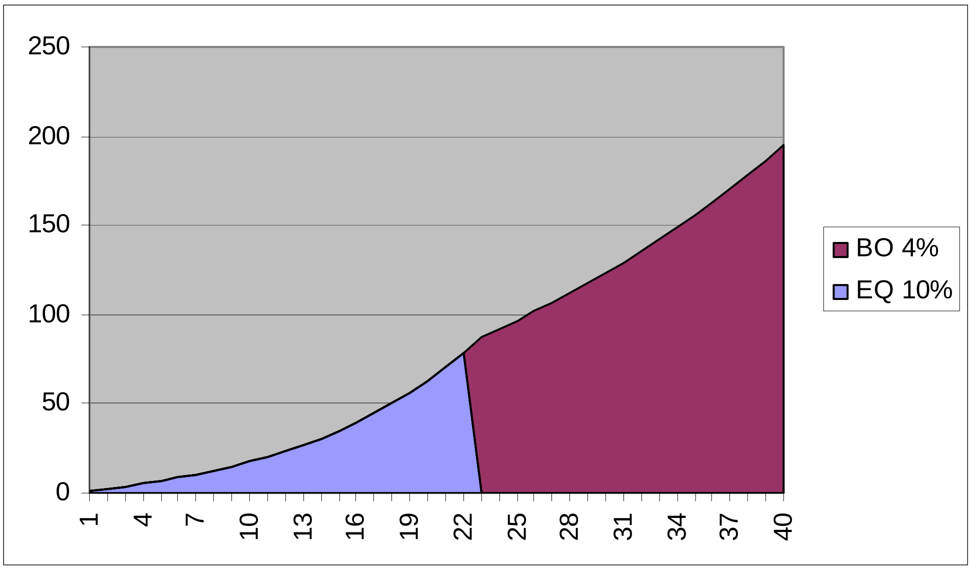
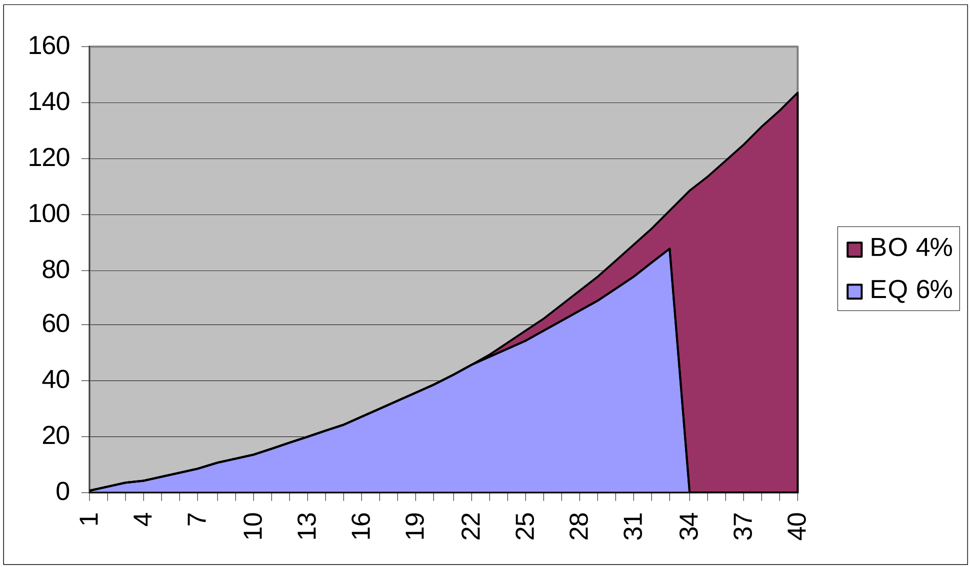
Appendix 1 -With "switch strategy (SC=23)", three different cases

Figure A.



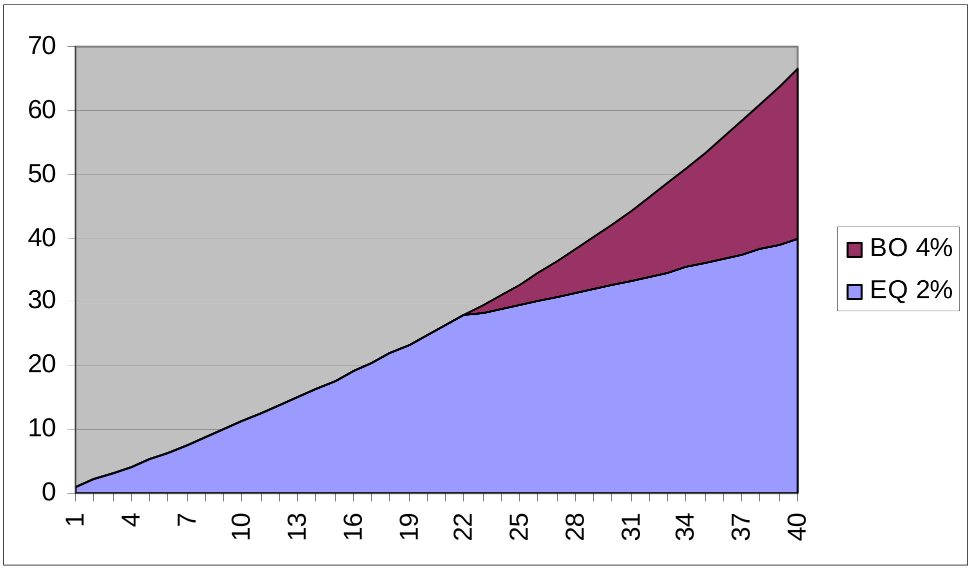
Return on equities is very high, so the switch will occur immediately SC=23.

Figure B.



The return on equities is not too high, you need some extra time to switch from equities to bonds.

Figure C.



Return on equities is too low in this case, the switch from the equity fund to bonds will not be made before retirement.

Appendix B - Linear regression

Figure D. The linear regression for SMS at for the years 23 to 40.

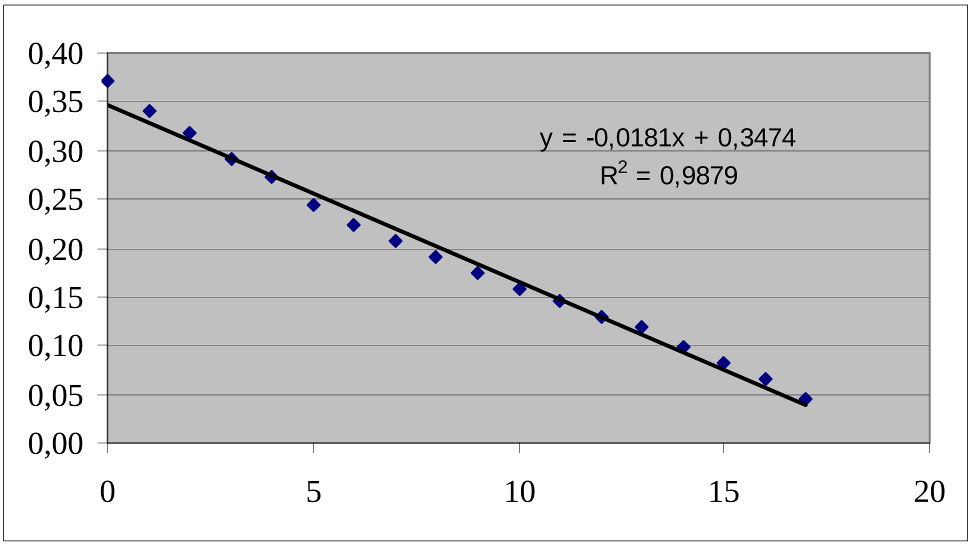
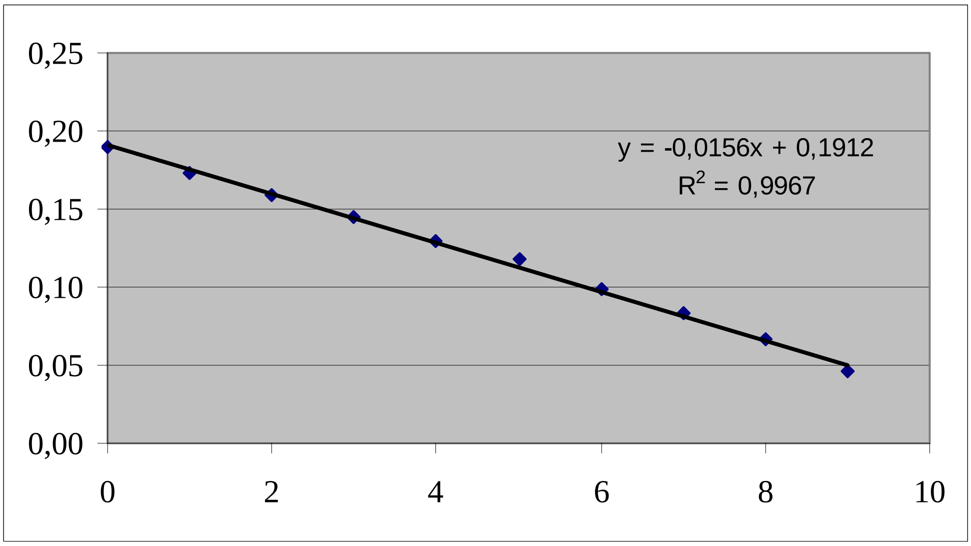


Figure E. The linear regression line for SMS for the years 31 to 40.



Appendix C - Discounted Buffer

Table A. The discounted buffer in comparison with the original buffer

Switch strategy	SC=23 with discounted buffer	SC=23 with buffer
Mean	163,77	167,8
Standard deviation	68,7	70,8
Downside deviation	50,8	52,5
Mean shortfall from the F_I^{TAR}	43,4	45,8
P($F_R^{TOT} < F_R^{TAR}$)	35,8%	34,7%
$P\left(F_R^{TOT} < F_R^{TAR} \mid SF < 41\right)$	10,0%	5,2%
Value at Risk 95%	66,5	66,4
Value at Risk 75%	117,4	114,2

Table B.

Switch strategy	SC=31 with discounted buffer	SC=31 with buffer
Mean	186,5	187,6
Standard deviation	105,0	105,1
Downside deviation	55,8	56,4
Mean shortfall from the F_I^{TAR}	48,1	49,0
P($F_R^{TOT} < F_R^{TAR}$)	33,3%	32,7%
$P\left(F_R^{TOT} < F_R^{TAR} \mid SF < 41\right)$	5,7%	4,0%
Value at Risk 95%	60,9	60,9
Value at Risk 75%	119,3	117,7

Table C.

Switch strategy	SC=23-31 with discounted buffer	SC=23-31 with buffer
Mean	165,0	169,2
Standard deviation	69,2	71,4
Downside deviation	53,6	56,8
Mean shortfall from the F_I^{TAR}	45,0	48,9
P($F_R^{TOT} < F_R^{TAR}$)	34,0%	32,1%
$P\left(F_R^{TOT} < F_R^{TAR} \mid SF < 41\right)$	10,8%	5,8%
Value at Risk 95%	61,9	60,9
Value at Risk 75%	122,7	121,2