# Internal vs. External Risk Measures: How Capital Requirements Differ in Practice

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*Abstract:*

We compare capital requirements derived by tail conditional expectation (TCE) with those derived by tail conditional median (TCM) and find that there is no clear-cut relationship between these two measures in empirical data. Our results highlight the relevance of TCM as a robust alternative to TCE, especially for regulatory control.

*JEL Classification*: G10, G11, G23, G29

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## 1. Introduction

Value-at-risk (VaR) or tail conditional expectation (TCE)? Since the publication of Artzner et al. [3], a great deal of iterature has appeared that discusses the drawbacks of VaR as compared to the benefits of TCE or conditional value-at-risk (CVaR). Specifically, Artzner et al. [3] define a set of axioms and call risk measures coherent if they satisfy these axioms. One of the axioms is subadditivity, which basically means that "a merger does not create extra risk" (Artzner et al. [3]). The authors show that VaR does not always satisfy the subadditivity axiom and thus conclude that VaR is not suitable for risk management. An alternative that satisfies the subadditivity axiom and is flexible enough to handle optimization procedures is CvaR. A very recent stream of literature focuses on the pros and cons VaR and TCE, in an attempt to discover which is best.

Only recently has the question of how to choose the best objective-oriented risk measure arisen in literature (see Heyde et al. [8]). The first question that needs to be answered is: For what "audience" is the risk measure intended—equity shareholders, regulatory/legal agencies, or internal management? There is no reason to believe that one, single risk measure will satisfy the needs of these different parties. In particular, some risk measures may be suitable for internal risk management but not for external regulatory agencies, and vice versa. For internal risk control, for example, coherent and convex risk measures are preferable because of their subadditive properties. However, for external risk measures, a different set of properties might be more appropriate, including consistency in implementation, which means robustness. In such a case, the so-called natural risk statistics, including the tail conditional median (TCM) as a special case, display more robustness than tail conditional expectation (see Cont et al. [4]).

The choice of risk measure is of great practical importance: It determines the minimum reserve for the computation of margin requirements in financial trading, insurance risk premiums, regulatory deposit requirements, among many others. The level of capital requirement is crucial to all profitability ratios that relate company profit to the underlying capital basis. A different risk measure may lead to lower (higher) capital requirement and thus to a higher (lower) profitability of the firm. Thus, a key question for practitioners is whether, given that the confidence level $\alpha$ is fixed, the choice of either TCE or TCM will lead to different capital requirements? And, if so, which measure will result in the most restrictive capital requirements?

The aim of this paper is to answer these questions. We thus empirically analyze a number of different risk positions involving different asset classes and different time horizons. Our main findings are: (1) TCE and TCM lead to different capital requirements, but there is no clear-cut relationship between TCE and TCM; (2) TCE is on average about 10% larger than TCM, but (3) in about 10% of all cases, TCM is higher than TCE, indicating that in these situations extreme tails may have very different behavior. The results are robust considering risk positions from different asset classes as well as different time horizons.

Our findings support the Heyde et al. [8] proposal to implement objective-oriented risk measures. In fact, choosing the most appropriate risk measure based on what it is expected to represent is not only correct as a normative approach, but should also be done in practice, since internal and external measures lead to different results.

The remainder of the paper is organized as follows. Section 2 discusses the properties desirable for internal and external measures. Section 3 presents empirical results on the TCE and TCM mismatch. Section 4 concludes.

## 2. Objectives of Risk Measures: Internal and External Risk Measures

Firms and regulators have different perspectives on risk management. Jarrow and Purnanandam [7] term these different perspectives as "capital budgeting" and "capital determination," respectively. The firm's goal is to achieve a portfolio allocation that maximizes its risk/return tradeoff, subject to any regulatory capital requirements. In contrast, taking the firm portfolio allocation as fixed, the regulator's goal is to determine how much capital the firm must have in order to limit the consequences of losses within a given time. Heyde et al. [8] stress this point from an axiomatic viewpoint, focusing on the different axioms that are desirable to each involved party. In their scheme, risk measures tailored to firm's goals are called "internal" measures, and those that are more in line with the regulator's goal are called "external" measures. For a deeper discussion, see Heyde et al. [8].

### 2.1 Internal Risk Measures: Tail Conditional Expectation

In the early 1990s, due to a number of spectacular company bankruptcies attributed to the inappropriate use of derivatives and a lack of sufficient internal controls, a new method of measuring risk called value-at-risk (VaR) was developed by regulators. Originally VaR was intended to measure the risks in derivatives markets, but it has become widely used to measure all kinds of financial risks, primarily market and credit risks, and even as a tool for active internal risk management.

In their seminal paper, Artzner et al. [3] point out VaR's lack subadditivity. The authors introduce a new family of subadditive measures called "coherent risk measures." One of these is the tail conditional expectation (TCE). More precisely, the TCE at level $\alpha \in (0\%,\ 100\%)$ is defined by:

$$TCE_{\alpha} = \text{mean of the } \alpha\text{-tail distribution of the loss } X.$$

If the distribution of $X$ is continuous, then

$$TCE_{\alpha}(X) = E\left[X \middle| X \geq VaR_{\alpha}(X)\right] \triangleright$$

For discrete distributions, if one appropriately defines quantiles for discrete random variables (see Acerbi and Tasche [1]), then $TCE_{\alpha}$ is the sample tail conditional expectation.

It is well known that expectation is a data-sensitive central measure and, consequently so is TCE. It follows that TCE is strongly influenced by model assumptions regarding the heaviness of tail distributions. Its appropriateness for use as an external risk measure by regulators is questionable. In fact, each firm has developed its own financial model and tail heaviness has become a subjective issue (see Heyde et al [8], pp. 12 ff.). However, even though TCE is not an appropriate regulatory or external risk measure, it has the potential to be an excellent internal risk measure. Since TCE is also a convex risk measure, it is widely used in stochastic convex optimization for determining optimal asset allocation.

### 2.2 External Risk Measure: Tail Conditional Median

Heyde et al. [8] discuss desirable axioms for an external risk measure. Subadditivity is relaxed only to comonotonic data. Comonotonic subadditivity is consistent with the prospect theory of risk in psychology. The new data-based class of risk measures introduced is called "natural risk statistics" and a representation theorem is given (see Ahmed et al. [2] for alternative proofs). A special case of natural risk measures is provided by tail conditional median (TCM). More precisely, the TCM at level $\alpha$ is defined by:

$$TCM_\alpha = \text{median of the } \alpha\text{-tail distribution of the loss } X,$$

or

$$TCM_\alpha(X) = median\left[X \middle| X \geq VaR_\alpha(X)\right].$$

This representation holds for both discrete and continuous distributions. If *X* is continuous, TCM can be rewritten in terms of VaR, since

$$TCM_\alpha(X) = VaR_{\frac{1+\alpha}{2}}(X).$$

As mentioned by Heyde et al. [8], and contrary to conventional wisdom, the above equation makes it evident that VaR, at a higher level, can also incorporate tail information. For example, if one wants a central measure of the loss beyond the 95% level, VaR at 97.5% is a possibility because it gives the tail conditional median at 95% level. Note that this holds only for continuous distributions and not in the discrete case (which is the case in real risk management using empirical data). However, also in the discrete case, VaR at 97.5% can be used as an approximation of TCM at 95%. As a median, TCM is a robust central measure and thus particularly suitable as an external risk measure. The empirical results on the robustness of conditional median presented by Ogryczak and Zawadzki [9] are further confirmation of the measure's theoretical attractiveness.

### 2.3 Subadditivity and Robustness: Conflicting Requirements

TCE and TCM are both natural statistical measures, but TCE is less robust than TCM. Cont et al. [4] show that it is only TCE's subadditivity property that makes it less robust, i.e., the property of being less sensitive to the tail distribution.

Note that there is no consensus in literature on the necessity of requiring subadditivity. Although VaR may violate the subadditivity property in the center of the distributions, Daníelsson et al. [5] state that this violation is merely of a technical nature, at least if one focuses on the tail regions most relevant to risk management. Indeed, the authors show that VaR is subadditive in the tail regions, provided that the tails in the joint distribution are not extremely fat. Since asset returns with extremely fat tails are not often found in financial markets but very easy to identify once spotted, Daníelsson et al. [5] argue that they can be treated as special cases in financial modeling. They also carry out simulations showing that VaR is indeed subadditive when $\alpha \in [95\%,\ 99\%]$, that is, for most practical applications. In conclusion, we do not believe it is necessary to avoid using VaR, even for internal purposes, because of its strong robustness and subadditivity in the confidence regions of financial interest.

## 3. Empirical Analysis

### 3.1. Data

In this section we analyze the empirical properties of tail conditional median as compared to those of tail conditional expectation. From a practical point of view, the most important thing to discover is whether these different risk measures lead to different results. An answer of “yes” will have important implications for risk management. If the answer comes up “no,” the discussion becomes academic. However, as we will show in the following, the TCE vs. TCM debate turns out to be important for both practitioners and academics.

We consider three data sources that incorporate both different return distribution characteristics and various time horizons.

(1) We consider returns of the 500 stocks contained in the S&P 500 and the S&P 500 index itself. We have return data available from the Datastream database for the period January 1990 to December 2004. We consider these data both on a daily and on a monthly basis.

(2) We analyze 1,374 mutual funds with monthly returns from January 1996 to December 2005. We choose a group of equity-oriented mutual funds previously analyzed in another context (Eling [6]) so as to improve comparability with the stock data presented under item (1) above. The data originate from the Datastream database. The consideration of mutual funds is different from the analysis of stocks in two aspects. First, mutual funds are portfolios of stocks and thus more diversified than the individual stocks. Second, using mutual fund data provides a natural way to incorporates transaction costs into our analysis. The mutual fund data are net of all fees, while the stock data do not incorporate any transaction costs.

(3) We also look at hedge fund data provided by the Center for International Securities and Derivatives Markets (CISDM). Our database contains 205 hedge funds reporting monthly returns, again net of all fees, for the period of January 1996 to December 2005. The consideration of hedge fund data provides further insights in our analysis because it is well known that the returns of hedge funds are very skewed and have much heavier tails than traditional investments. Hedge funds might therefore be the asset class with the most extreme tails in financial markets.

Table 1 contains descriptive statistics on the return distributions of the stocks, mutual funds, and hedge funds. The table sets out mean, median, standard deviation, minimum, and maximum of the first four moments of the return distribution (mean value, standard deviation, skewness, and excess kurtosis). For example, the standard deviation in Row 5 means that across the 500 stocks, the standard deviation of the returns has a mean of 10.60% (second column in Row 5) with a standard deviation of 4.57% (fourth column in Row 5). Note that all data in Table 1 are monthly numbers.

Insert Table 1 here

The individual stocks generate the highest returns on average, but such is accompanied by a very high risk in terms of standard deviation. The returns of most of the individual stocks are positively skewed and have positive excess kurtosis. On the basis of the Jarque-Bera test, the assumption of normally distributed stock returns must be rejected for 56.83% at the 1% significance level. The diversified mutual funds provide a lower return with a lower risk. The lower return might be driven by the transaction costs that are considered within the mutual fund returns, but not for the individual stocks. The lower standard deviation illustrates the risk reduction through portfolio diversification. As indicated, the returns of hedge funds are more skewed and have fatter tails than do those of the mutual funds. It is thus not surprising to see that the Jarque-Bera rejection rate is far higher for hedge funds than it is for mutual funds.

### 3.2 Discussion of Results

#### (1) S&P 500 Index and 500 Stocks

We split the analysis of the S&P 500 data into two steps. In the first step, we consider only the S&P 500 index, which aggregates the performance of all 500 stocks; this analysis is comparable to the analysis presented in Heyde et al. [8], p. 25. In the second step, the 500 stocks contained in the index are analyzed on a disaggregated level; here, we present average values and other statistics about the 500 stocks.

Our analysis of the S&P 500 index covers 3,915 daily index returns in the period January 1990 to December 2004. Table 2 shows the tail conditional expectation (TCE) and tail conditional mean (TCM) for different confidence levels. To highlight the difference between TCE and TCM, we present the difference between TCE and TCM in Column 4 and the relative difference between TCE and TCM in Column 5.

Insert Table 2 here

We confirm Heyde et al.'s [8], p. 25, results concerning the S&P 500 daily losses. In almost all cases, TCE is larger than TCM, denoting the presence of loss fat tails. The differences can be quite significant; for example, at a confidence level of 90%, TCE is 11.23% higher than TCM. Note, however, that at a confidence level of 99.9%. TCM is larger than TCE. This switching behavior illustrates that the magnitude of fatness is less for extreme catastrophic events. The findings in Table 2 are the only ones in this paper that are based on a single index; all the tables that follow present average values for a large number of stocks or funds.

To achieve a broader view of the differences between TCE and TCM, we now consider the 500 stocks that are listed in the S&P 500 index (3,915 daily returns in the period from January 1990 to December 2004). Table 3 contains for different confidence levels $\alpha$ the corresponding values of TCE and TCM, their variation, and their percentage variation.

Insert Table 3 here

Comparing results of Table 3 with those of Table 2, we see that TCE is always larger than TCM. The percentage variation is greater than that revealed for the single index; it is about 12% for confidence levels $\alpha$ above 95%. It thus seems that the TCE always leads to higher capital requirements than the TCM.

Figure 1 shows the frequency distribution of TCE minus TCM for the 500 stocks in the S&P. The horizontal axis shows the difference between TCE and TCM and the vertical axis shows the number of funds. The left part of the figure shows the results for $\alpha$ = 90%; the right part for $\alpha$ = 95%. Beneath the figure are some descriptive statistics on the distribution (min, 25% quantile, median, 75% quantile, max, mean, range (=max-min), percentage of negative values). The mean value corresponds to the TCE minus TCM value presented in Table 3.

Insert Figure 1 here

The frequency distribution illustrates that the difference between TCE and TCM is positive in almost 100% of the cases. While at a confidence level of 95%, TCE is always equal to or larger than TCM, at a confidence level of 90% we find one case where the difference is negative. Again, these finding indicate that in the case of stocks, TCE nearly always leads to higher capital requirements than does TCM.

We now turn a look at monthly data, which is important to the because for certain risk positions there are no daily data available (e.g., in case of hedge funds). In Table 4 we consider monthly data for the 500 stocks in the S&P. For this purpose, we transformed the 3,915 daily returns into 180 monthly returns.

Insert Table 4 here

Again, TCE is larger than TCM, although the difference between them is relatively low for confidence levels higher than 95%. Note that the perfect matching that occurs at confidence levels higher than 99% is not surprising because we consider the lowest 1% values within 180 monthly returns, meaning that we look at a very small number of data points and thus mean and median are often very similar.

Figure 2 shows the frequency distribution of TCE minus TCM. The horizontal axis shows the difference between TCE and TCM and the vertical axis shows the number of funds.

Insert Figure 2 here

An important aspect of Figure 2 is the much higher number of negative values for TCE minus TCM than were seen in Figure 1. This finding would indicate that for about 6.2% of the 500 stocks, the capital requirement derived by way of TCM would be larger than those derived by way of TCE. It is also noteworthy that the variation (range=max-min) is much larger here, e.g., with $\alpha$ = 95% the range is 0.1290, whereas it is only 0.0200 with the daily stock data.

**(2) Mutual Funds**

In Table 5, we consider data from 1,374 mutual funds reporting monthly net of fee returns in the period from January 1996 to December 2005. The table contains for each confidence level the corresponding values of TCE and TCE, their variation, and their percentage variation.

Insert Table 5 here

Again, TCE is larger than TCM and, also again, the difference between the two is negligible at confidence levels greater than 95%. Figure 3 shows the frequency distribution of TCE minus TCM. The horizontal axis shows the difference between TCE and TCM and the vertical axis shows the number of funds.

Insert Figure 3 here

In Figure 3, we see even an even greater number of negative values for TCE minus TCM than was the case in Figures 1 and 2. For example, The variation increases, e.g., for $\alpha$ = 95%, the range is 0.1610 compared to 0.0200 with the stocks on a daily basis (Figure 1) and 0.1290 with stocks on a monthly basis (Figure 2).

**(3) Hedge Funds**

In Table 6, we consider 205 hedge funds reporting monthly net of fee returns in the period from January 1996 to December 2005.

Insert Table 6 here

Table 6 reveals that the differences between TCE and TCM are greater in regard to hedge funds than they are with regard to either stocks or mutual funds. At a confidence level of $\alpha$ = 95%, TCE is 13.05% higher than TCM; for mutual funds, this difference was only 6.22%. Again, the perfect congruence of results at confidence levels higher than 98.5% is due to purely technical reasons because we are only able to consider 120 monthly returns. Although it would be helpful to extend the time series to more than 120 data points, doing so is not possible as no such data are available. Hedge funds publish return information only once every a month and historical hedge fund returns (prior to 1996) suffer from backfilling bias (see Eling [6]), among other problems. In short, there simply are no other hedge fund data available for risk management purposes.

Again, we look at the data frequency in order to capture the variability of the values around their means. Figure 4 shows the frequency distribution of the difference, TCE minus TCM, for the 205 hedge funds. The horizontal axis shows the difference between TCE and TCM and the vertical axis shows the number of funds.

Insert Figure 4 here

We find that, on average, the TCE leads to higher capital requirements than the TCM; however, in about 7.32% of the cases TCM is higher than TCE. Although the hedge funds returns analyzed are not normally distributed and exhibit heavy tails, the distribution of TCE minus TCM is not too different in the case of hedge funds that was observed for mutual funds.

**4. Conclusion**

If the decision as to which risk measure is most appropriate is based on axiomatical grounds, the choice will be guided by the purpose of risk management, that is, different methods will be better depending on if it is internal or external risk management that is desired. Among the internal risk measures, the most common is the TCE, whereas the TCM is among the most popular external measures. A relevant question posed by practitioners is whether the choice of risk measure has an effect on the resulting capital requirements. The aim of this paper was to answer this question. Empirical investigations on possible mismatching between TCE and TCM were performed for different asset classes (stocks and the S&P 500 index; mutual funds; hedge funds) and different time horizons (daily and monthly data).

Heyde et al. [8], p. 23, based on one set of data (the S&P 500 index), conclude that the differences between TCE and TCM are highly significant, both theoretically and practically We performed a broader empirical analysis; our results (1) confirm the Heyde et al. findings and (2) emphasize the relevance of TCM. We find that TCE and TCM can lead to very different capital requirements. On average, and at standard confidence levels, TCE is about 10% larger than TCM. However, there is no clear-cut relationship between TCE and TCM; depending on the tails, we find that TCM can be higher than TCE. The findings support the proposal to use objective-oriented risk measures and underline the relevance of TCM as a robust natural risk statistic.

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| Time-Series Analysis | Cross-Sectional Analysis (across funds) Mean | Median | Standard Deviation | Minimum | Maximum |
|---|---|---|---|---|---|
| *A. 500 stocks from the S&P 500 (monthly data, January 1990 to December 2004)* | | | | | |
| *JB rejection: 56.83% (66.67%) at 1% (5%) level* | | | | | |
| Mean value (%) | 1.65 | 1.37 | 1.09 | -0.61 | 6.77 |
| Standard deviation (%) | 10.60 | 9.39 | 4.57 | 2.65 | 28.97 |
| Skewness | 0.21 | 0.17 | 0.67 | -2.43 | 7.03 |
| Excess kurtosis | 2.22 | 1.19 | 4.56 | -0.42 | 65.90 |
| *B. 1,314 mutual funds (monthly data, January 1996 to December 2005)* | | | | | |
| *JB rejection: 19.84% (26.73%) at 1% (5%) level* | | | | | |
| Mean value (%) | 0.53 | 0.49 | 1.19 | –9.52 | 9.79 |
| Standard deviation (%) | 4.70 | 4.50 | 2.43 | 0.06 | 29.31 |
| Skewness | –0.29 | –0.32 | 0.76 | –9.50 | 9.38 |
| Excess kurtosis | 0.76 | 0.11 | 4.35 | –7.19 | 100.83 |
| *C. 205 hedge funds (monthly data, January 1996 to December 2005)* | | | | | |
| *JB rejection: 80.00% (84.39%) at 1% (5%) level* | | | | | |
| Mean value (%) | 1.05 | 0.97 | 0.54 | -0.52 | 4.36 |
| Standard deviation (%) | 4.14 | 3.46 | 3.14 | 0.23 | 25.49 |
| Skewness | -0.07 | 0.00 | 1.49 | -6.40 | 7.87 |
| Excess kurtosis | 6.14 | 2.59 | 10.41 | -1.33 | 81.90 |

Table 1: Descriptive statistics for stocks, mutual funds, and hedge funds

| Alpha | TCE | TCM | TCE-TCM | (TCE-TCM)/TCE |
|---|---|---|---|---|
| 99.90% | 0.0610 | 0.0631 | -0.0022 | -3.58% |
| 99.50% | 0.0406 | 0.0364 | 0.0042 | 10.38% |
| 99.00% | 0.0346 | 0.0306 | 0.0039 | 11.36% |
| 98.50% | 0.0315 | 0.0285 | 0.0030 | 9.54% |
| 98.00% | 0.0294 | 0.0267 | 0.0028 | 9.42% |
| 97.50% | 0.0279 | 0.0254 | 0.0026 | 9.16% |
| 97.00% | 0.0266 | 0.0242 | 0.0024 | 9.06% |
| 96.50% | 0.0255 | 0.0233 | 0.0022 | 8.75% |
| 96.00% | 0.0246 | 0.0223 | 0.0022 | 9.15% |
| 95.50% | 0.0238 | 0.0217 | 0.0021 | 8.84% |
| 95.00% | 0.0231 | 0.0210 | 0.0021 | 9.11% |
| 90.00% | 0.0183 | 0.0163 | 0.0021 | 11.23% |
| 85.00% | 0.0154 | 0.0137 | 0.0017 | 11.00% |
| 80.00% | 0.0133 | 0.0111 | 0.0023 | 16.88% |
| 70.00% | 0.0104 | 0.0082 | 0.0022 | 21.24% |
| 60.00% | 0.0083 | 0.0062 | 0.0021 | 25.22% |
| 50.00% | 0.0067 | 0.0045 | 0.0022 | 32.76% |

Table 2: TCE and TCM for S&P 500 index (daily data)

| Alpha | TCE | TCM | TCE-TCM | (TCE-TCM)/TCE |
|---|---|---|---|---|
| 99.90% | 0.1536 | 0.1455 | 0.0081 | 5.12% |
| 99.50% | 0.1011 | 0.0894 | 0.0117 | 10.99% |
| 99.00% | 0.0833 | 0.0730 | 0.0104 | 11.94% |
| 98.50% | 0.0745 | 0.0652 | 0.0093 | 12.11% |
| 98.00% | 0.0685 | 0.0600 | 0.0085 | 12.15% |
| 97.50% | 0.0643 | 0.0562 | 0.0080 | 12.21% |
| 97.00% | 0.0608 | 0.0531 | 0.0077 | 12.33% |
| 96.50% | 0.0581 | 0.0507 | 0.0074 | 12.48% |
| 96.00% | 0.0557 | 0.0485 | 0.0071 | 12.55% |
| 95.50% | 0.0536 | 0.0467 | 0.0069 | 12.70% |
| 95.00% | 0.0518 | 0.0451 | 0.0067 | 12.85% |
| 90.00% | 0.0406 | 0.0348 | 0.0058 | 14.14% |
| 85.00% | 0.0343 | 0.0290 | 0.0053 | 15.55% |
| 80.00% | 0.0300 | 0.0249 | 0.0051 | 16.87% |
| 70.00% | 0.0239 | 0.0192 | 0.0047 | 19.78% |
| 60.00% | 0.0193 | 0.0149 | 0.0045 | 23.27% |
| 50.00% | 0.0122 | 0.0076 | 0.0046 | 42.92% |

Table 3: TCE and TCM for 500 stocks from the S&P 500 (daily data)

| Alpha | TCE | TCM | TCE-TCM | (TCE-TCM)/TCE |
|---|---|---|---|---|
| 99.90% | 0.3017 | 0.3017 | 0.0000 | 0.00% |
| 99.50% | 0.3017 | 0.3017 | 0.0000 | 0.00% |
| 99.00% | 0.2754 | 0.2754 | 0.0000 | 0.00% |
| 98.50% | 0.2582 | 0.2500 | 0.0083 | 3.24% |
| 98.00% | 0.2442 | 0.2355 | 0.0087 | 3.67% |
| 97.50% | 0.2334 | 0.2224 | 0.0110 | 4.93% |
| 97.00% | 0.2245 | 0.2136 | 0.0109 | 4.99% |
| 96.50% | 0.2164 | 0.2047 | 0.0118 | 5.55% |
| 96.00% | 0.2090 | 0.1962 | 0.0128 | 6.08% |
| 95.50% | 0.2031 | 0.1895 | 0.0135 | 6.66% |
| 95.00% | 0.2012 | 0.1874 | 0.0138 | 6.82% |
| 90.00% | 0.1628 | 0.1465 | 0.0163 | 10.00% |
| 85.00% | 0.1397 | 0.1229 | 0.0168 | 12.04% |
| 80.00% | 0.1228 | 0.1061 | 0.0167 | 13.71% |
| 70.00% | 0.0978 | 0.0816 | 0.0161 | 16.59% |
| 60.00% | 0.0785 | 0.0630 | 0.0155 | 20.18% |
| 50.00% | 0.0623 | 0.0472 | 0.0151 | 25.12% |

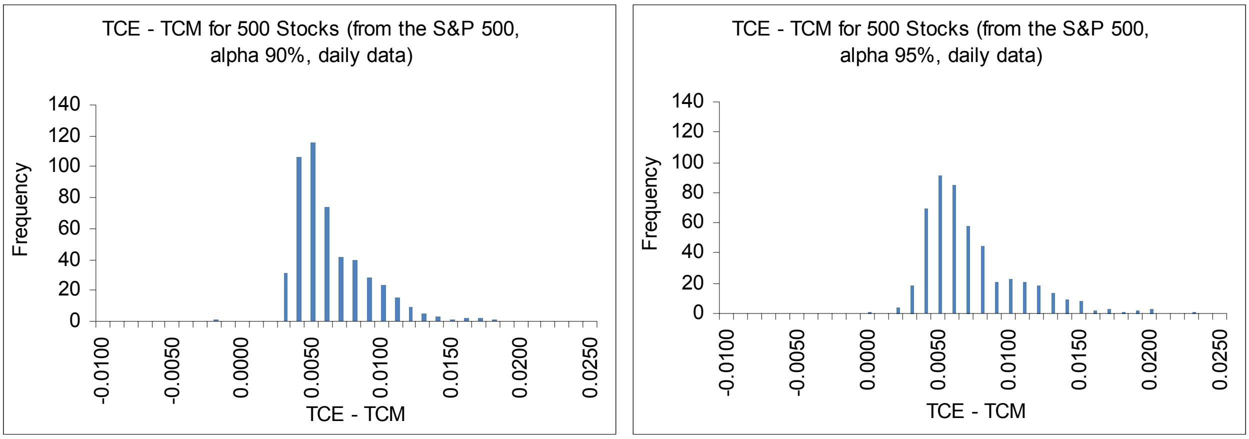
Table 4: TCE and TCM for 500 stocks from the S&P 500 (monthly data)

| Alpha | TCE | TCM | TCE-TCM | (TCE-TCM)/TCE |
|---|---|---|---|---|
| 99.90% | 0.1647 | 0.1647 | 0.0000 | 0.00% |
| 99.50% | 0.1647 | 0.1647 | 0.0000 | 0.00% |
| 99.00% | 0.1457 | 0.1457 | 0.0000 | 0.00% |
| 98.50% | 0.1457 | 0.1457 | 0.0000 | 0.00% |
| 98.00% | 0.1344 | 0.1267 | 0.0077 | 5.74% |
| 97.50% | 0.1344 | 0.1267 | 0.0077 | 5.74% |
| 97.00% | 0.1262 | 0.1194 | 0.0069 | 5.44% |
| 96.50% | 0.1195 | 0.1120 | 0.0075 | 6.29% |
| 96.00% | 0.1195 | 0.1120 | 0.0075 | 6.29% |
| 95.50% | 0.1139 | 0.1068 | 0.0071 | 6.22% |
| 95.00% | 0.1139 | 0.1068 | 0.0071 | 6.22% |
| 90.00% | 0.0909 | 0.0826 | 0.0083 | 9.19% |
| 85.00% | 0.0769 | 0.0673 | 0.0096 | 12.47% |
| 80.00% | 0.0668 | 0.0569 | 0.0099 | 14.76% |
| 70.00% | 0.0521 | 0.0418 | 0.0103 | 19.77% |
| 60.00% | 0.0324 | 0.0223 | 0.0101 | 31.11% |
| 50.00% | 0.0121 | 0.0015 | 0.0106 | 87.29% |

Table 5: TCE and TCM for 1,374 mutual funds (monthly data)

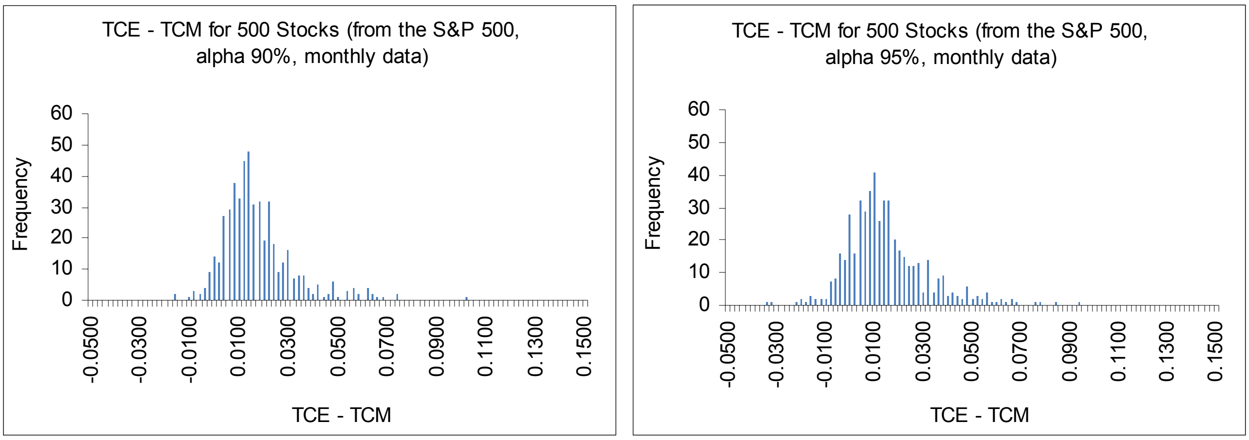
| Alpha | TCE | TCM | TCE-TCM | (TCE-TCM)/TCE |
|---|---|---|---|---|
| 99.90% | 0.1330 | 0.1330 | 0.0000 | 0.00% |
| 99.50% | 0.1330 | 0.1330 | 0.0000 | 0.00% |
| 99.00% | 0.1110 | 0.1110 | 0.0000 | 0.00% |
| 98.50% | 0.1110 | 0.1110 | 0.0000 | 0.00% |
| 98.00% | 0.0980 | 0.0889 | 0.0091 | 9.32% |
| 97.50% | 0.0980 | 0.0889 | 0.0091 | 9.32% |
| 97.00% | 0.0893 | 0.0805 | 0.0087 | 9.79% |
| 96.50% | 0.0829 | 0.0724 | 0.0105 | 12.68% |
| 96.00% | 0.0829 | 0.0724 | 0.0105 | 12.68% |
| 95.50% | 0.0778 | 0.0676 | 0.0101 | 13.05% |
| 95.00% | 0.0778 | 0.0676 | 0.0101 | 13.05% |
| 90.00% | 0.0590 | 0.0499 | 0.0091 | 15.38% |
| 85.00% | 0.0487 | 0.0396 | 0.0091 | 18.65% |
| 80.00% | 0.0415 | 0.0334 | 0.0082 | 19.66% |
| 70.00% | 0.0315 | 0.0236 | 0.0079 | 24.97% |
| 60.00% | 0.0242 | 0.0169 | 0.0073 | 30.24% |
| 50.00% | 0.0182 | 0.0111 | 0.0070 | 38.70% |

Table 6: TCE and TCM for 205 hedge funds (monthly data)



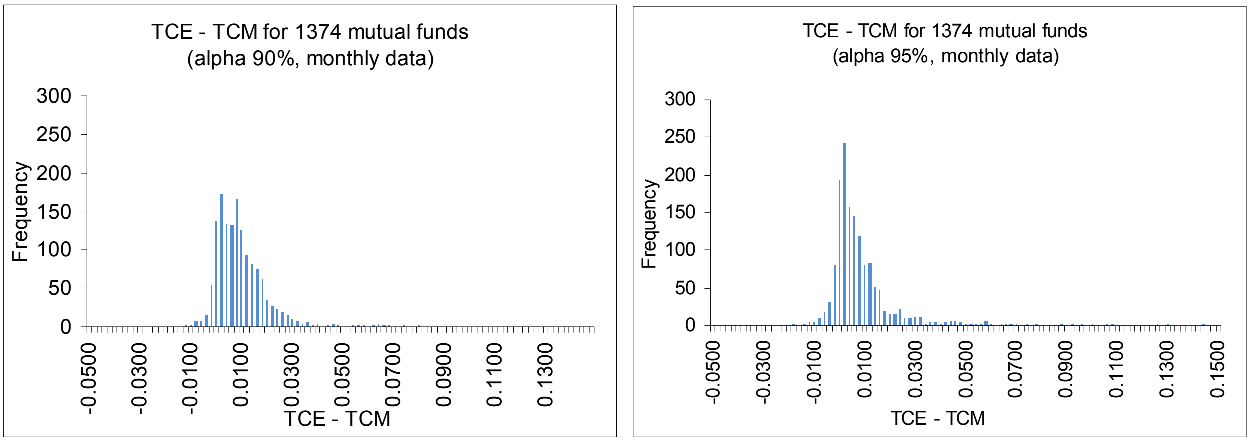

| Alpha | Min | 25% | 50% (Median) | 75% | Max | Mean | Range | % negative |
|---|---|---|---|---|---|---|---|---|
| 90.00% | -0.0028 | 0.0039 | 0.0050 | 0.0071 | 0.0172 | 0.0058 | 0.0200 | 0.20% |
| 95.00% | 0.0000 | 0.0044 | 0.0058 | 0.0081 | 0.0221 | 0.0067 | 0.0221 | 0.00% |

Figure 1: Frequency distribution of TCE minus TCM (500 stocks, daily data)



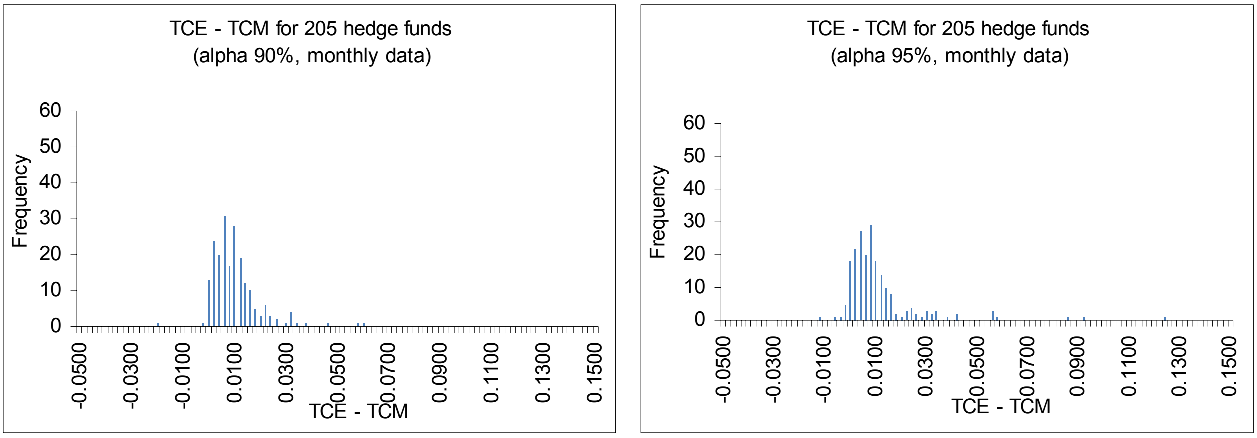

| Alpha | Min | 25% | 50% (Median) | 75% | Max | Mean | Range | % negative |
|---|---|---|---|---|---|---|---|---|
| 90.00% | -0.0169 | 0.0074 | 0.0132 | 0.0217 | 0.1014 | 0.0163 | 0.1182 | 6.20% |
| 95.00% | -0.0350 | 0.0035 | 0.0108 | 0.0211 | 0.0940 | 0.0138 | 0.1290 | 16.00% |

Figure 2: Frequency distribution of TCE minus TCM (500 stocks, monthly data)




| Alpha | Min | 25% | 50% (Median) | 75% | Max | Mean | Range | % negative |
|---|---|---|---|---|---|---|---|---|
| 90.00% | -0.0124 | 0.0015 | 0.0066 | 0.0126 | 0.0797 | 0.0084 | 0.0920 | 15.54% |
| 95.00% | -0.0187 | 0.0001 | 0.0037 | 0.0095 | 0.1423 | 0.0071 | 0.1610 | 23.76% |

Figure 3: Frequency distribution of TCE minus TCM (1,374 mutual funds, monthly data)




| Alpha | Min | 25% | 50% (Median) | 75% | Max | Mean | Range | % negative |
|---|---|---|---|---|---|---|---|---|
| 90.00% | -0.0209 | 0.0035 | 0.0078 | 0.0118 | 0.0588 | 0.0091 | 0.0798 | 7.32% |
| 95.00% | -0.0139 | 0.0022 | 0.0063 | 0.0113 | 0.1228 | 0.0101 | 0.1367 | 12.68% |

Figure 4: Frequency distribution of TCE minus TCM (205 hedge funds, monthly data)